Introduction to bootstrapping using discount factors

An *n*-period discount factor is the present value of 1 unit of currency (£1 or $1) that is payable at the end of period *n*. Essentially it is the present value relationship expressed in terms of £1. If $d(n)$ is the *n*-year discount factor, then the five-year discount factor at a discount rate of 6% is given by

$$d(5) = \frac{1}{(1+0.06)^5} = 0.747258.$$

The set of discount factors for every time period from 1 day to 30 years or longer is termed the *discount function*. Discount factors may be used to price any financial instrument that is made up of a future cash flow. For example what would be the value of £103.50 receivable at the end of six months if the six-month discount factor is 0.98756? The answer is given by:

0.98756 x 103.50 = 102.212.

In addition discount factors may be used to calculate the future value of any present investment. From the example above, £0.98756 would be worth £1 in six months' time, so by the same principle a present sum of £1 would be worth

$1 / d(0.5) = 1 / 0.98756 = 1.0126$

at the end of six months.

It is possible to obtain discount factors from current bond prices. Assume an hypothetical set of bonds and bond prices as given in table 1 below, and assume further that the first bond in the table matures in precisely six months time (these are semi-annual coupon bonds).

Coupon	Maturity date	Price
7%	07-Jun-01	101.65
8%	07-Dec-01	101.89
6%	07-Jun-02	100.75
6.50%	07-Dec-02	100.37

Table 1 Hypothetical set of bonds and bond prices

Taking the first bond, this matures in precisely six months' time, and its final cash flow will be 103.50, comprised of the £3.50 final coupon payment and the £100 redemption payment. The price or present value of this bond is 101.65, which allows us to calculate the six-month discount factor as:

$$d(0.5)\times 103.50 = 101.65$$

which gives *d*(0.5) equal to 0.98213.

From this first step we can calculate the discount factors for the following six-month periods. The second bond in table 1, the 8% 2001 has the following cash flows:

- £4 in six month's time
- £104 in one year's time

The price of this bond is 101.89, which again is the bond's present value, and this is comprised of the sum of the present values of the bond's total cash flows. So we are able to set the following:

$$101.89 = 4\times d(0.5) + 104\times d(1).$$

However we already know *d*(0.5) to be 0.98213, which leaves only one unknown in the above expression. Therefore we may solve for *d*(1) and this is shown to be 0.94194.

If we carry on with this procedure for the remaining two bonds, using successive discount factors, we obtain the complete set of discount factors as shown in table 2. The continuous function for the two-year period from today is shown as the discount function, at figure 1.

Coupon	Maturity date	Term (years)	Price	$d(n)$
7%	07-Jun-01	0.5	101.65	0.98213
8%	07-Dec-01	1.0	101.89	0.94194
6%	07-Jun-02	1.5	100.75	0.92211
6.50%	07-Dec-02	2.0	100.37	0.88252

Table 2 Discount factors calculated using bootstrapping technique

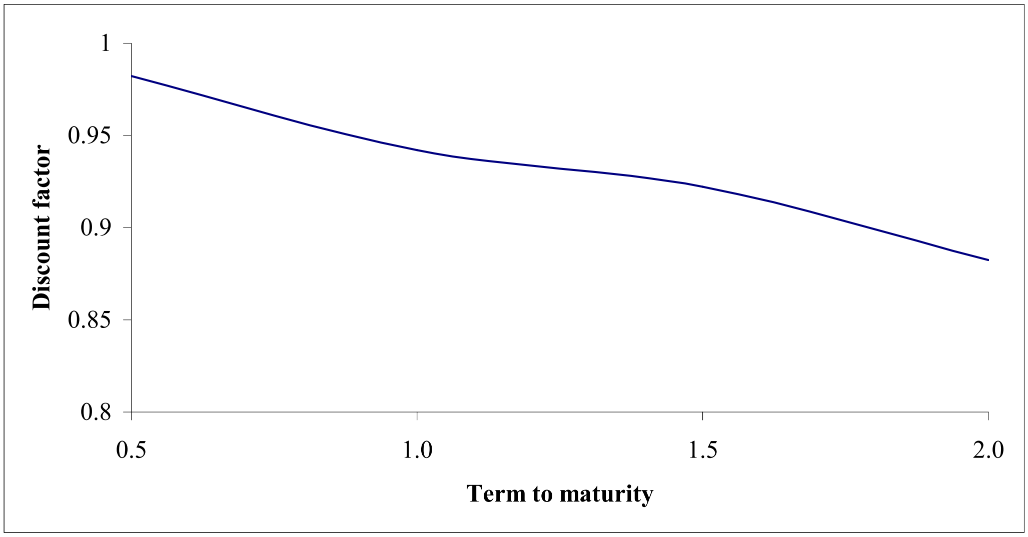

Figure 1 Hypothetical discount function

This technique, which is known as *bootstrapping*, is conceptually neat but problems arise when we do not have a set of bonds that mature at precise six-month intervals. In addition liquidity issues connected with specific individual bonds can also cause complications. However it is still worth being familiar with this approach.

Note from figure 1 how discount factors decrease with increasing maturity: this is intuitively obvious, since the present value of something to be received in the future diminishes the further into the future we go.