Inflation derivatives: introduction

One of the latest developments in derivatives markets are inflation-linked derivatives, or, simply, inflation derivatives. The first examples were introduced into the market in 2001. They arose out of the desire of investors for real, inflation-linked returns and hedging rather than nominal returns. Although index-linked bonds are available for those wishing to have such returns, as we've observed in other asset classes, inflation derivatives can be tailor-made to suit specific requirements. Volume growth has been rapid during 2003, as shown in Figure 9.4 for the European market.

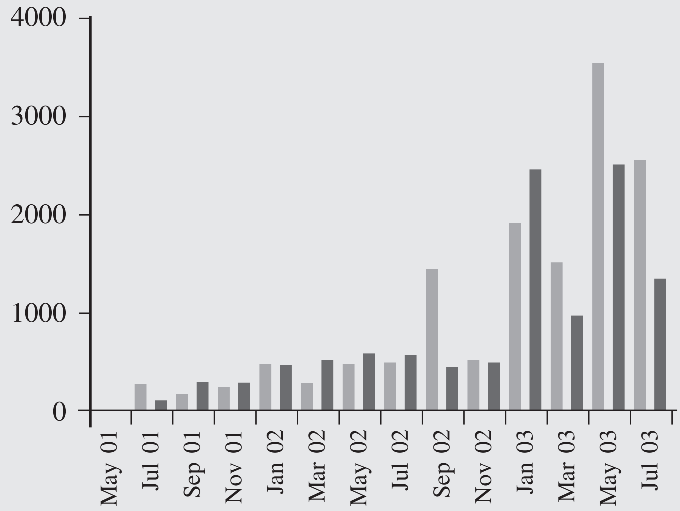

Figure 9.4 Inflation derivatives volumes, 2001-2003
Source: ICAP

The UK market, which features a well-developed index-linked cash market, has seen the largest volume of business in inflation derivatives. They have been used by market-makers to hedge inflation-indexed bonds, as well as by corporates who wish to match future liabilities. For instance, the retail company Boots plc added to its portfolio of inflation-linked bonds when it wished to better match its future liabilities in employees' salaries, which were assumed to rise with inflation. Hence, it entered into a series of inflation derivatives with Barclays Capital, in which it received a floating-rate, inflation-linked interest rate and paid nominal fixed-rate interest rate. The swaps ranged in maturity from 18 to 28 years, with a total notional amount of £300 million.

While this is an example of a pension fund that wishes to *receive* inflation-linked returns, corporates that receive revenues linked to inflation often wish to hedge this by *paying* inflation. For instance, utility companies often have pricing structures dictated by regulatory authorities, who may cap price increases that can be passed to customers. Such companies are natural issuers of index-linked bonds, and may therefore be interested in paying inflation in exchange for a fixed-rate return. Another UK-market corporate deal involved National Air Traffic Services, which paid inflation in a £200 million swap (again with Barclays Capital) in 2002. For market-making banks who take on this exposure, the usual hedge is with an index-linked bond such as UK gilts or Treasury TIPS.

Inflation swaps may be priced in a number of ways. The most common method involves determining prices from index-linked bonds using the bootstrapping approach. Assuming a zero-coupon swap, the market-maker calculates the inflation rate to use in the swap from the difference between the yields of index-linked and conventional bonds of the same maturity. This is the breakeven inflation rate we discussed earlier in the chapter. Some banks use asset-correlation models to price the swaps, such as the type presented in Jarrow and Yildirim (2003), which uses three stochastic factors to price inflation-linked bonds and swaps.

Inflation-indexed derivatives

Inflation-indexed derivatives, also known as inflation-linked derivatives *or inflation derivatives*, have become widely traded instruments in the capital markets in a relatively short space of time. They are traded generally by the same desks in investment banks that trade inflation-linked sovereign bonds, who use these instruments for hedging as well as to meet the requirements of clients such as hedge funds, pension funds and corporates. They are a natural development of the inflation-linked bond market.

Inflation derivatives are an additional means by which market participants can have an exposure to inflation-linked cash flows. They can also improve market liquidity in inflation-linked products, as an earlier generation of derivatives did for interest-rates and credit risk. As flexible OTS products, inflation derivatives offer advantages over cash products in certain circumstances. They provide:

- an ability to tailor cash flows to meet investors' requirements;
- a means by which inflation-linked exposures can be hedged;
- an instrument via which relative value positions can be put on across cash and synthetic markets;
- a building block for the structuring of more complex and hybrid products.

The inflation derivatives market in the UK was introduced after the introduction of the gilt repo market in 1996. In most gilt repo trades, index-linked (IL) gilts could be used as collateral; this mean that both IL and conventional gilts could be used as hedging tools against positions in inflation derivatives. In the euro area, IL derivatives were introduced later but experienced significant growth during 2002-2003. The existence of a sovereign IL bond market can be thought of as a necessary precursor to the development of IL derivatives, and although there is no reason why this should be the case, up to now this has been the case. The reason for this is probably because such a cash market suggests that investors are aware of the attraction of IL products, and wish to invest in them. From a market in IL bonds then develops a market in IL swaps, which are the most common IL derivatives. The IL bond market also provides a ready reference point form which IL derivatives can be priced.

Market instruments

We describe first some common inflation derivatives, before considering some uses for hedging and other purposes. We then consider IL derivatives pricing.

Inflation-linked bond swap

This is also known as a *synthetic index-linked bond*. It is a swap with the following two cashflow legs:

- pay (receive) the cashflows on a government IL bond;
- receive (pay) a fixed or floating cashflow.

This converts am existing conventional fixed- or floating-rate investments into inflation-linked investments. An example of such a swap is given below.

IL bond swap

Nominal	100,000,000
Start date	15 March 2004
Maturity term	15 March 2009
Bank receives	Six-month Euribor flat [+ spread], semi-annual, act/360 or Fixed rate coupon *x*%, annual 30/360
Bank pays	Real coupon of *y*% y * [HICP (p – 3) / (HICP (s – 3))] * daycount * notional annual 30/360 On maturity: Notional * max {0%, [HICP (m – 3) / HICP (s – 3) – 1]}

The symbols in the formulae above are

p	payment date
s	start date
m	maturity date
HCIP	Harmonised Index of Consumer Prices

The "minus 3" in the formula for HCIP refers to a three-month lag for indexation, common in euro sovereign IL bond markets.

The swap is illustrated at Figure 9.5.

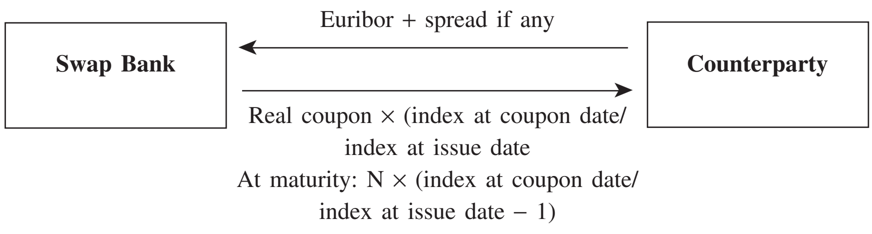

Figure 9.5 Synthetic index-linked bond

Year-on-year inflation swap

This swap is commonly used to hedge issues of IL bonds. The swap is comprised of:

- pay (receive) an index-linked coupon, which is a fixed rate component plus the annual rate of change in the underlying index
- receive (pay) Euribor or Libor, plus a spread if necessary.

With these swaps, IL leg is usually set at a floor of 0% for the annual change in the underlying index. This guarantees the investor a minimum return of the fixed rate coupon.

This swap is also known as a *pay-as-you-go swap*. It is shown at Figure 9.6.

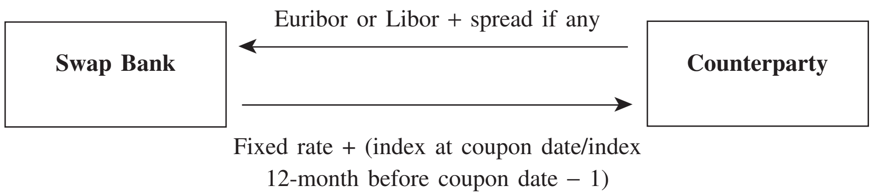

Figure 9.6 Year-on-year inflation swap

Tips swap

The Tips swap is based on the structure of US Tips securities. It pays a periodic fixed rate on an accreting notional amount, together with an additional one-off payment on maturity. This payout profile is identical to many government IL bonds. They are similar to synthetic IL bonds described above.

Tips swaps are commonly purchased by pension funds and other long-dated investors. They may prefer the added flexibility of the IL swap market compared to the cash IL bond market. Figure 9.7 shows the Tips swap.

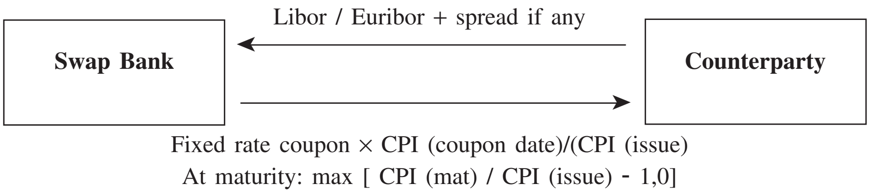

Figure 9.7 Illustration of a Tips swap

Breakeven swap

This is also known as a zero-coupon inflation swap or zero-coupon swap. It allows the investor to hedge away a breakeven exposure. Compared to IL swaps such as the synthetic bond swap, which hedge a real yield exposure, the breakeven swap has both cashflow legs paying out on maturity. The legs are:

- the total return on the inflation index
- a compounded fixed breakeven rate.

This structure enables IL derivative market makers to hedge their books. It is illustrated at Figure 9.8.

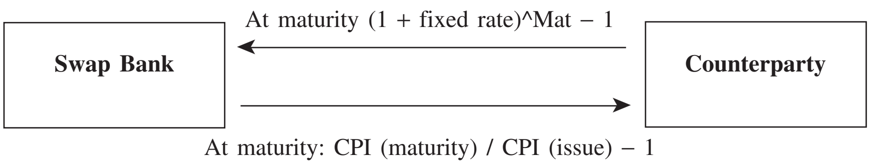

Figure 9.8 Breakeven inflation swap

Real annuity swap

A real-annuity swap is used to hedge inflation-linked cashflows where this applies for payments such as rental streams, lease payments, project finance cashflows and so on. It enables market participants who pay or receive such payments to replace the uncertainty of the future level of these cashflows with a fixed rate of growth. The swap is written on the same notional amount for both legs, but payout profiles differ as follows:

- the index-linked leg of the swap compounds its payments with the rate of change of the index
- the fixed leg of the swap compounds its payments at a pre-specified fixed rate.

These swaps are one of the most commonly traded. The fixed rate quoted for the swap provides a ready reference point against which to compare expected future rates of inflation. So for instance, if a bank is quoting for a swap with a fixed rate of 3.00%, and an investor believes that inflation rates will not rise above 3.00% for the life of the swap, then it will receive "fixed" (here meaning a fixed rate of growth) and pay inflation-linked on the swap.

The inflation term structure and pricing inflation derivatives

An inflation term structure is a necessary prerequisite to the pricing of inflation derivatives. It is constructed using the same principles we discussed in chapters 3 and 4. Previously, to construct this curve we would have used IL bond prices as the set of market yields used as inputs to the curve. Now however we can also use the prices of IL derivatives. As with other markets, the derivative prices are often preferred to cash prices for two reasons; one, we can use a continuous set of prices rather than have to rely on available bond maturities, and secondly, there is usually greater liquidity in the OTC market.

In the case of IL products, the indexation element is not in fact a true picture, but rather a picture based on a lag of three, six or seven months. This lag needs to be taken into account when constructing our curve.

The forward index value I at time T from time t ($t < T$) is given by

$$I(t,T) = \frac{I(t)P_r\ (t,T)}{P_r\ (t,T)} \tag{9.12}$$

where

$I(t)$ is the index value at time t

$P_r(t,T)$ is the price at time t of a real zero-coupon bond of par value 1 maturing at T

$P_n(t,T)$ is the price at time t of a nominal zero-coupon bond of par value 1 maturing at T

Using Equation 9.12 we can build a forward inflation curve provided we have the values of the index at present, as well as a set of zero-coupon bond prices of required credit quality. Following standard yield curve analysis, we may build the term structure from forward rates and therefore imply the real yield curve, or alternatively we may construct the real curve and project the forward rates. However if we are using inflation swaps for the market price inputs, the former method is preferred because IL swaps are usually quoted in terms of a forward index value.

The curve can be constructed using standard bootstrapping techniques.

Inflation derivatives can be priced reasonably accurately once the inflation term structure is constructed. However some practitioners use stochastic models in pricing such products to account for the volatility surfaces. That is, they model the volatility of inflation as well. The recent literature describes such methods. For instance, Bezooyen *et al* (1997), Hughston (1998) and Jarrow and Yildirm (2003) suggest an approach based on that described by Amin and Jarrow (1991). This assumes that suitable proxies for the real and nominal term structures are those of foreign and the domestic economies. In other words, the foreign exchange rate captures the information required to model the two curves. We describe this approach here.

We assume that the index follows a lognormal distribution, and we use normal models for the real and nominal forward rates. For the index we then have

$$\mathrm{d}I(t) = I(t)[\mu_I(t)\mathrm{d}t + \sigma(t)\mathrm{d}W(t)] \tag{9.13}$$

and we have

$$\mathrm{d}F_n(t,T) = a_n(t,T)\mathrm{d}t + \sigma_n(t,T)\mathrm{d}W(t) \tag{9.14}$$

$$\mathrm{d}F_r(t,T) = a_r(t,T)\mathrm{d}t + \sigma_r(t,T)\mathrm{d}W(t) \tag{9.15}$$

for the nominal and real forward rate processes.

The dynamics of the zero-coupon bonds introduced earlier for Equation 9.13 are given by

$$\mathrm{d}(\log P_k(t,T)) = \left[r_k(t) - \int_t^T \alpha_k(t,u)\mathrm{d}u\right]\mathrm{d}t + \sum\nolimits_k(t,T)\mathrm{d}W(t) \tag{9.16}$$

and where $k = n,r$

$$\sum_k(t,T) = -\int \alpha_k(t,u)\mathrm{d}u$$
$$k = n,r$$

describes the zero-coupon bond volatilities. We use a one-factor model (see chapter 4) for each of the term structures and one for the index. Therefore $\mathrm{d}W(t)$ is a combined three-dimensional vector of three correlated Brownian or Weiner or processes, with a correlation of ρ. The volatility of each bond and the index is therefore also a three-element vector.

From the above, the price at time t of an option on the index struck at X and expiring at time T is given by

$$V_\phi^{Index}(t,T) = \phi[I(t)P_r(t,T)N(\phi h_1) - XP_n(t,T)N(\phi h_2)] \qquad (9.17)$$

where

$\phi = 1$ for a call option and $\phi = -1$ for a put option, and where

$$h_1 = \left(\log\left(\frac{I(t)P_r(t,T)}{P_n(t,T)X}\right) + \frac{V(t,T)^2}{2}\right)/V(t,T)$$

$$h_2 = h_1 - V(t,T)$$

and where

$$V(t,T)^2 = \int_t^T \sum_I(u,t)\cdot\rho\cdot\sum_I(u,T)\mathrm{d}u$$

and where we define

$$\sum_I(t,T) = \sum_n(t,T) - \sum_r(t,T) - \sigma_I(u) \quad .$$

The result above has been derived, in different forms, in all three references noted above. As with other options pricing models, it needs to be calibrated to the market before it can be used. Generally this will involve using actual and project forward inflation rates to fit the model to market prices and volatilities.

Applications

We now describe some common applications of IL derivatives.

Hedging pension liabilities

This is perhaps the most obvious application. Assume a life assurance company or corporate pension fund wishes to hedge its long-dated pension liabilities, which are linked to the rate of inflation. It may invest in sovereign IL bonds such as IL gilts, or in IL corporate bonds that are hedged (for credit risk purposes) with credit derivatives. However the market in IL bonds is not always liquid, especially in IL corporate bonds. The alternative is to buy a synthetic IL bond. This is structured as a combination of a conventional government bond and an IL swap, in which the pension fund pays away the bond coupon and receives inflation-linked payments.

The net cashflow leaves the pension fund receiving a stream of cash flow that are linked to inflation. The fund is therefore hedged against its liabilities. In addition, because the swap structure can be tailor-made to the pension fund's requirements, the dates of cashflows can be set up exactly as needed. This is an added advantage over investing in the IL bonds directly.

Portfolio restructuring using inflation swaps

Assume that a bank or corporate has an income stream that is linked to inflation. Up to now, it has been funded by a mix of fixed- and floating-rate debt. Say that these are floating-rate bank loans and fixed-rate bonds. However from an asset-liability management (ALM) point of view this is not optimal, because of the nature of a proportion of its income. It makes sense, therefore, to switch a part of its funding into an inflation-linked segment. This can be done using either of the following approaches:

- issue an IL bond;
- enter into an IL swap, with a notional value based on the optimum share of its total funding that should be inflation-linked, in which it pays inflation-linked cash flows and receives fixed-rate income.

The choice will depend on which approach provides cheapest funding and most flexibility.

Hedging a bond issue

Assume that a bank or corporate intends to issue an IL bond, and wishes to hedge against a possible fall in government IL bond prices, against which its issue will be priced. It can achieve this hedge using a IL gilt-linked derivative contract.

The bank or corporate enters into cash-settled contract for difference (CFD), which pays out in the event of a rise in government IL bond yields. The CFD has a term to maturity that ties in with the issue date of the IL bond. The CFD market maker has effectively shorted the government bond, from the CFD trade date until maturity. On the issue date, the market maker will provide a cash settlement if yields have risen. If yields have fallen, the IL bond issuer will pay the difference. However, this cost is netted out by the expected "profit" from the cheaper funding when the bond is issued. Meanwhile, if yields have risen and the bank or corporate issuer does have to fund at a higher rate, it will be compensated by the funds received by the CFD market maker.