Share Price Movements

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Share Price Movements

$$\mathbf{dS} = \mathbf{S}\,\mu\,\mathbf{dt} + \mathbf{S}\,\sigma\,\mathbf{dz} \cdots (\mathbf{1})$$

In continuous time or

$$\Delta\mathbf{S} = \mathbf{S}\,\mu\,\Delta\mathbf{t} + \mathbf{S}\,\sigma\,\Delta\mathbf{z} \cdots (\mathbf{2})$$

In discrete (measurable) time

Where:

dS or ΔS represents the change in the stock price

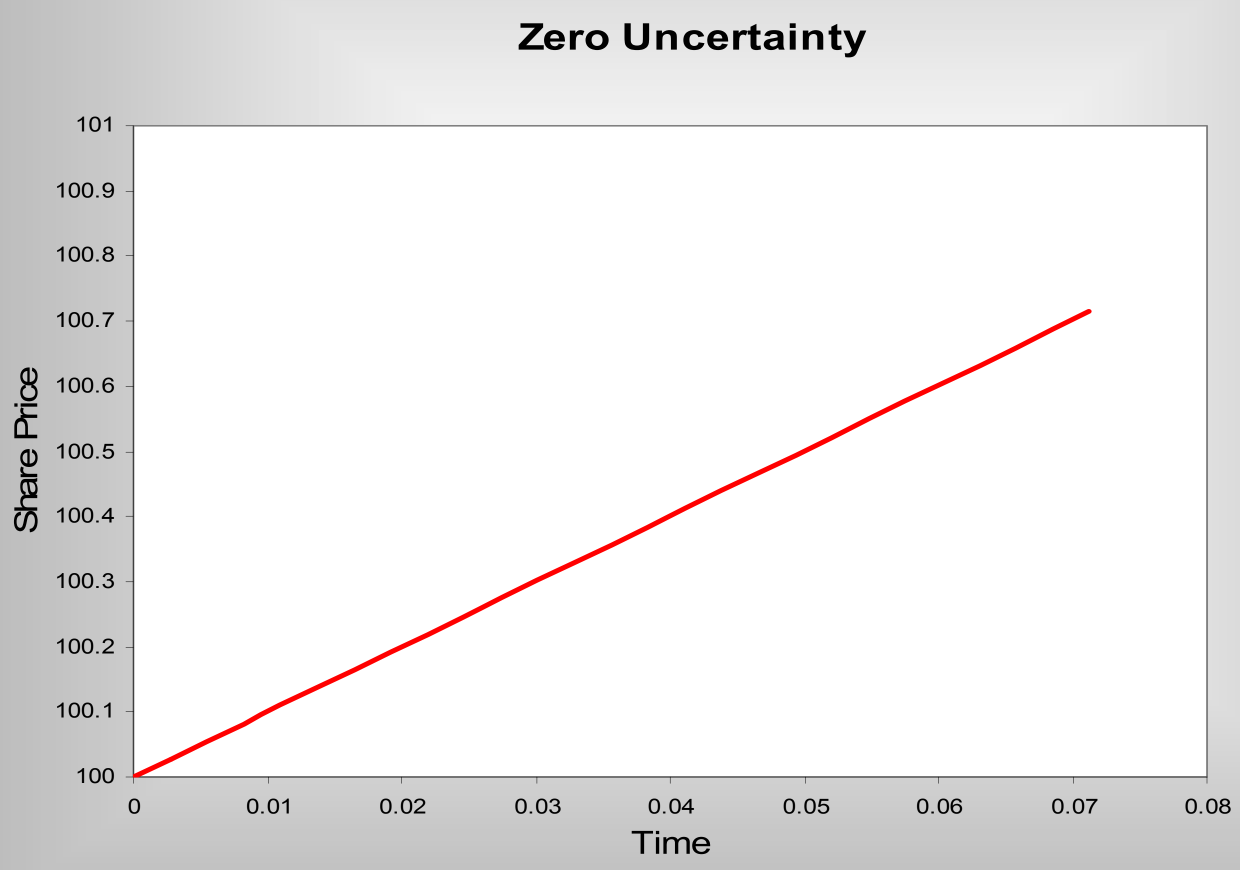
μ represents the mean return on the stock (the drift factor)

σ represents the stock's volatility

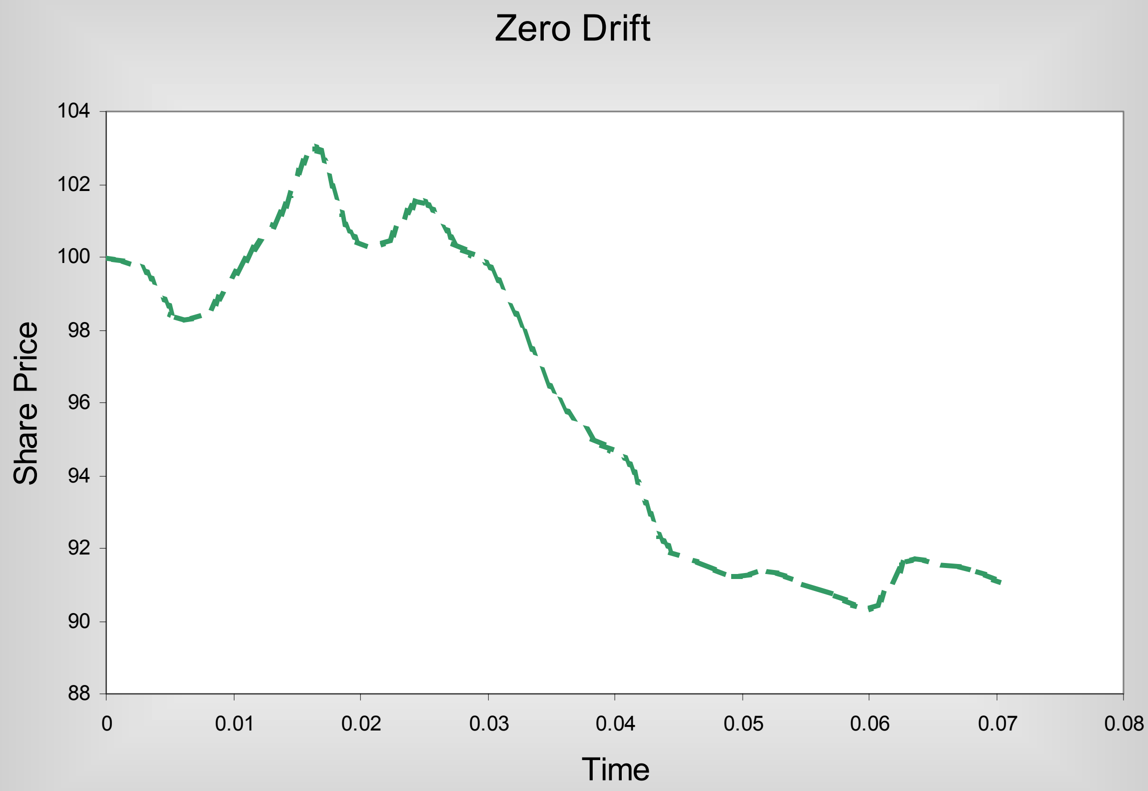
dt or Δt represents the change in time over which the change in stock price is measured

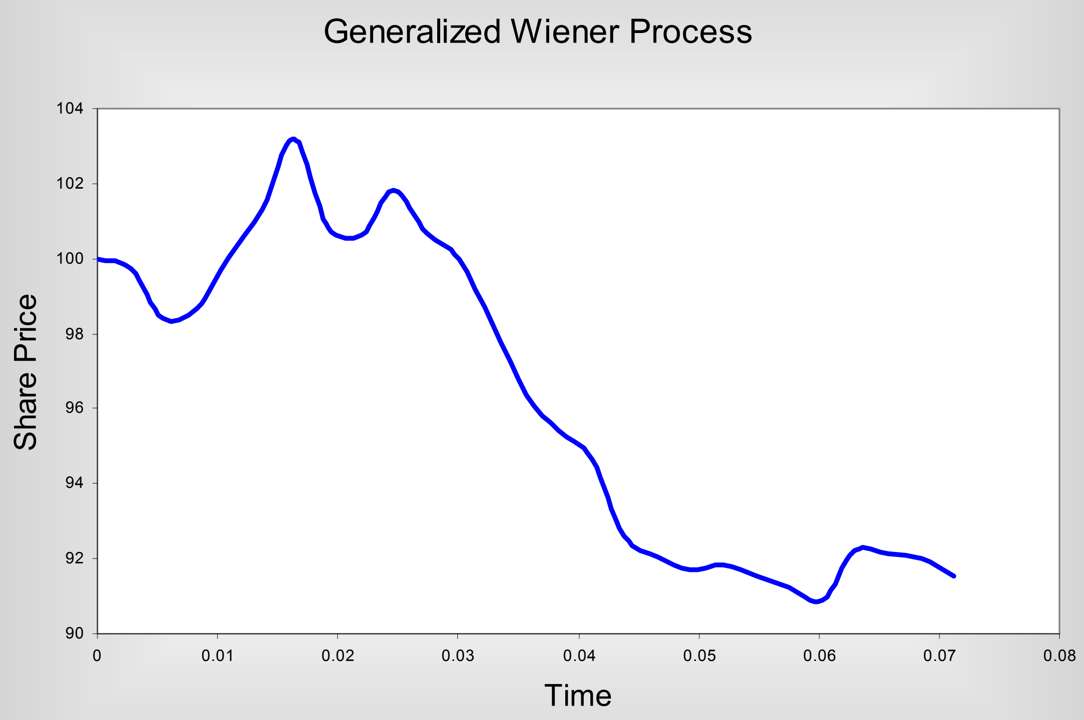
The term dz or Δz captures the uncertainty associated with the stock price movement.

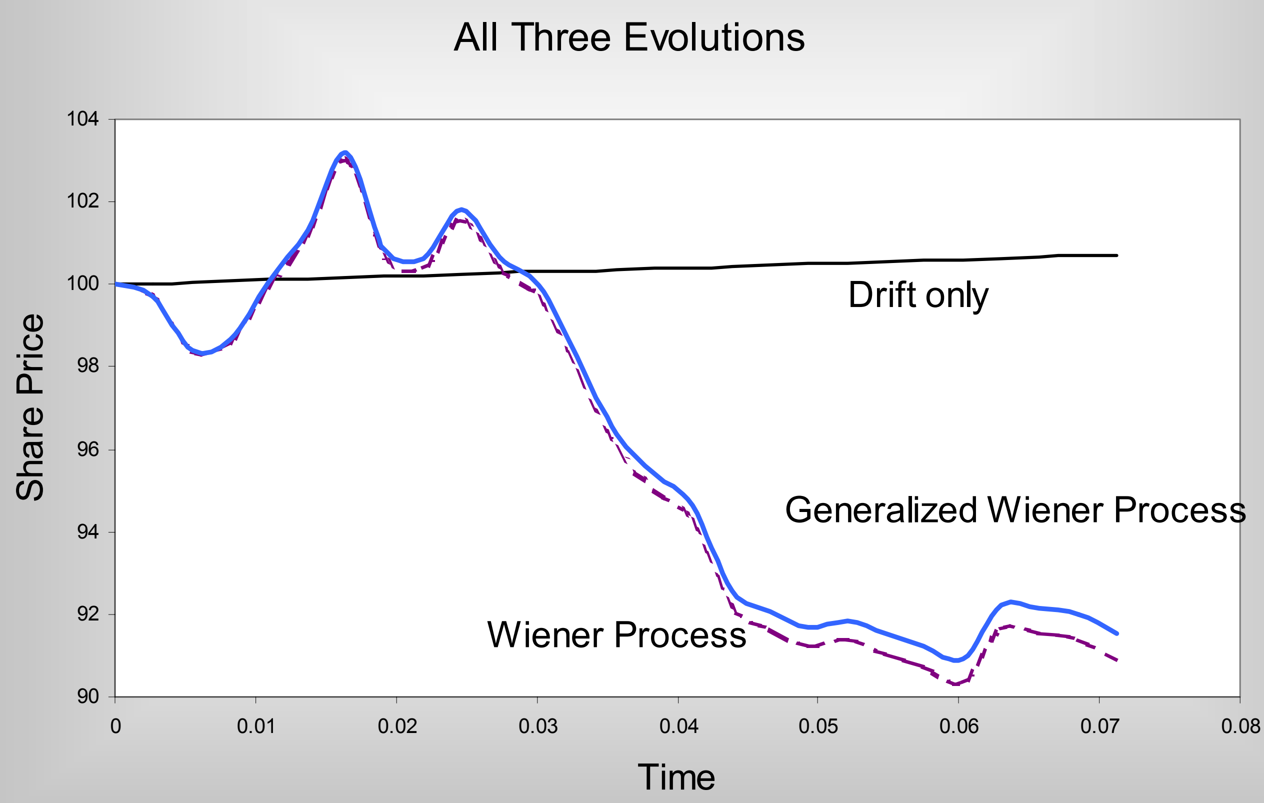
Equations (1) and (2) are examples of a *Generalized Wiener process.*



Zero Uncertainty
Share Price
101
100.9
100.8
100.7
100.6
100.5
100.4
100.3
100.2
100.1
100
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
Time


Zero Drift
Share Price
104
102
100
98
96
94
92
90
88
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
Time


Generalized Wiener Process
Share Price
104
102
100
98
96
94
92
90
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
Time



All Three Evolutions
Share Price
104
102
100
98
96
94
92
90
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
Time
Drift only
Generalized Wiener Process
Wiener Process

How are the Evolutions Calculated?

The tracks involving uncertainty have been calculated using Monte Carlo simulation.

A set of random numbers is generated to replace the Δz term in equation 2.

In generating this track a further assumption is used:

$$\Delta z = \varepsilon\sqrt{\Delta t} \cdots (3)$$

where ε is normally distributed (0, 1)

$\varepsilon \sim$ **N(0, 1) and**

$\Delta S/S \sim N(\mu\Delta t, \sigma(\Delta t)^{1/2})$

Processes like this can be used to obtain prices for exotic options.

Example

Simulate the daily track of a share whose starting value is 100, return 10% p.a., and volatility 20%p.a.

$$\Delta t = 1 - \text{day} = 0.00274$$

$$\text{daily return} = (0.01)(0.00274) = 0.000274$$

$$\text{daily volatility} = \sigma\sqrt{\Delta t} = (0.2)\left(\sqrt{0.00274}\right) = 0.010468$$

Taking a N(0, 1) random number (0.11656), the next day's simulated price is:

$$\Delta S + S = 100[(0.000274) + (0.010468)(0.11656)] + 100$$

$$\Delta S + S = S_1 = 0.1494 + 100$$

$$S_1 = 100.1494$$

Repeating this for several evolutions gives the following tracks:

Share Price Evolution	Share Price Changes	Random Numbers
100.0000	0.149415	0.11656
100.1494	-1.312036	-1.27768
98.8374	0.2797973	0.244257
99.1172	1.3515742	1.276474
100.4688	1.2878416	1.19835
101.7566	1.8739939	1.733133

Log_e share price evolutions

In the trinomial case trees were developed using $\log_e$ evolution rather than the underlying's value. In practice this is also done in simulations, too.

$$\ln S(t+\Delta t) = \ln S_t + \left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma\sqrt{\Delta t}\,\varepsilon$$

or $$S + \Delta S = S e^{\left[\left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma\sqrt{\Delta t}\,\varepsilon\right]}$$

Share Price Evolution	Share Price Evolution	Share Price Changes	Random Numbers
100	4.60517019	0.0011661	0.11656
100.11668	4.60633629	-0.013429	-1.27768
98.781215	4.59290745	0.0025028	0.244257
99.028757	4.59541029	0.0133081	1.276474
100.35545	4.60871836	0.0124903	1.19835
101.61678	4.62120864	0.0180884	1.733133

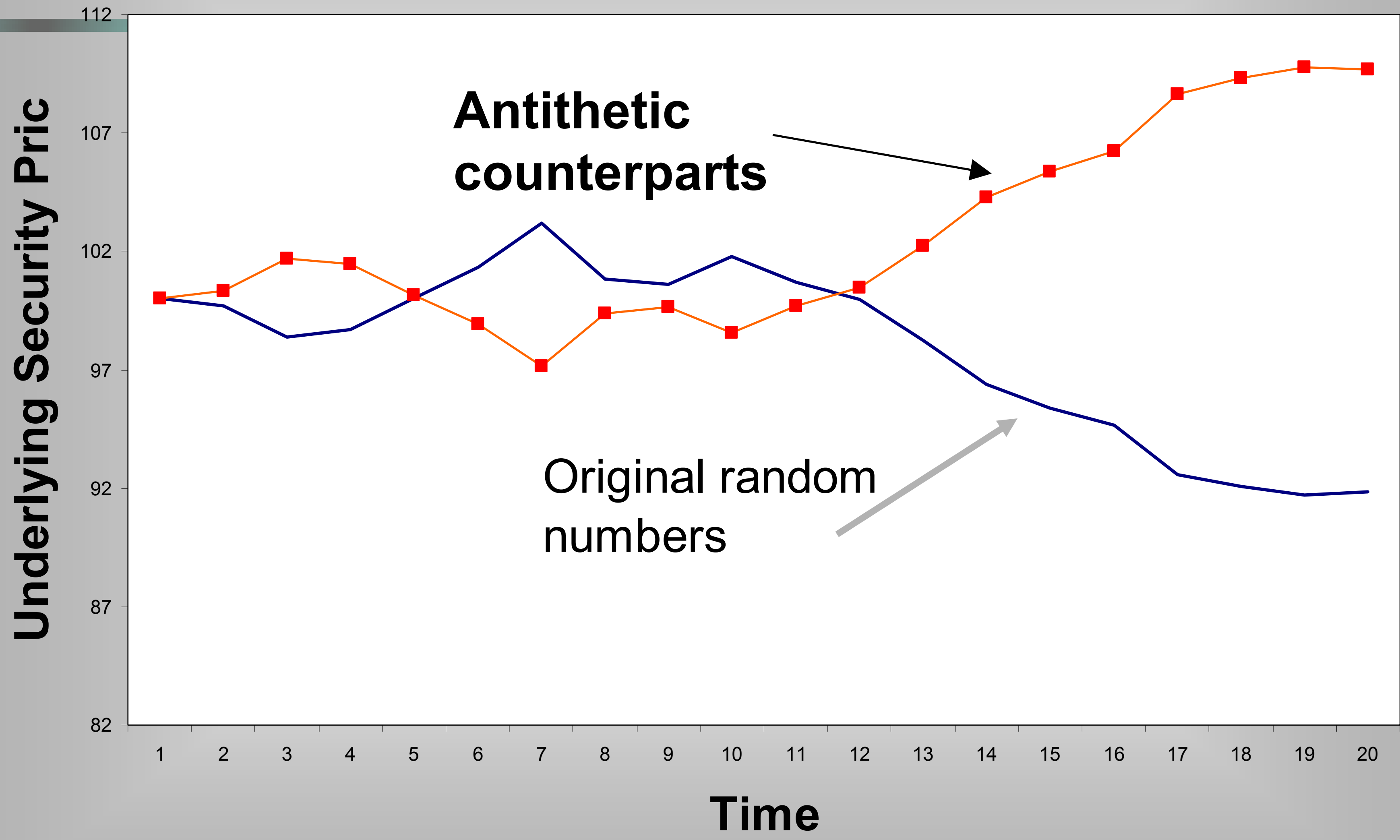
Problems with the basic model

Problems associated with the approach - constant variance, constant drift, slow.

<u>Antithetic variables:</u> can help to speed up the process by averaging outcomes of the mirror image tracks, i.e.

$$\mathbf{p_s} = \frac{\left(\mathbf{p}^{+} + \mathbf{p}^{-}\right)}{\mathbf{2}}$$

Antithetic Variables Approach



Hull approach:

$$p_a = p_{asv} - p_{bcv} + p_b$$

where:

p_a is the estimated option variable,

p_{asv} is the simulated stochastic variance option variable,

p_{bcv} is the simulated control variate option variable,

p_b is the known option variable value.

Clewlow and Strickland approach:

They use delta, gamma, and vega as control variates. In this way the hedge parameters are also obtained as a direct by product of the simulation. For a call option, for example the simulation proceeds in the normal way but for each loop the control variate (cv) is accumulated for use in the expiration formula:

$$C_T = \max\left[S_T - E,\ 0\right] + \left(\beta \cdot cv\right)$$

where: β is set at -1, cv is the control variate found as:

$$cv_t = cv_{t-1} + delta.\left(S_t - S_0 * drift\right)$$

The call option value at expiration will be the discounted average of all the payout values obtained from the M simulated tracks.

Bootstrapping:

Is a parameter free approach and permits the past data tracks to determine average future tracks.

All simulation methods can provide a measure of variance reduction by using the standard error formula:

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{M}}$$

Stochastic variance

$$\Delta S = S\mu\Delta t + S\sigma\Delta w$$

$$\Delta\sigma = \sigma\left[\frac{\theta^2}{2} - \beta\left(\ln\sigma - \bar{a}\right)\right]\Delta t + \theta\sigma\Delta z_1$$

where: $\Delta\sigma$ represents the change in volatility that occurs over the time period under consideration, $\bar{a}$ represents the mean reverting value for lnσ volatility to which the volatility each day, σ, is adjusting at a rate β, θ is the proportional volatility of the volatility Δz_1 is given by $\Delta z_1 = \varepsilon_1 \sqrt{\Delta t}$ where ε_1 is a random drawing from the N(0,1) distribution adjusted to reflect the volatility's mean drift and variance over the discretised time frame used in the simulations.

Following the work of Hofmann, Platen, and Schweizer (1992) a third equation was included to model daily adjustments to $\bar{a}$. This model takes the form:

$$\Delta \bar{\sigma} = \frac{1}{\lambda}\left(\sigma - \bar{\sigma} \right)\Delta t$$