The yield curve, and spot and forward interest rates
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In this primer we consider the zero-coupon or spot interest rate and the forward rate. We also look at the yield curve. Investors consider a bond yield and the general market yield curve when undertaking analysis to determine if the bond is worth buying; this is a form of what is known as *relative value* analysis. All investors will have a specific risk/reward profile that they are comfortable with, and a bond's yield relative to its perceived risk will influence the decision to buy (or sell) it.

We consider the different types of yield curve, before considering a specific curve, the zero-coupon or spot yield curve. Yield curve construction itself requires some formidable mathematics and is outside the scope of this book; we consider here the basic techniques only. Interested readers who wish to study the topic further may wish to refer to the author's book *Analysing and Interpreting the Yield Curve.*

B. THE YIELD CURVE

We have already considered the main measure of return associated with holding bonds, the *yield to maturity* or *redemption yield*. Much of the analysis and pricing activity that takes place in the bond markets revolves around the *yield curve*. The yield curve describes the relationship between a particular redemption yield and a bond's maturity. Plotting the yields of bonds along the term structure will give us our yield curve. It is important that only bonds from the same class of issuer or with the same degree of liquidity be used when plotting the yield curve; for example a curve may be constructed for gilts or for AA-rated sterling Eurobonds, but not a mixture of both.

In this section we will consider the yield to maturity yield curve as well as other types of yield curve that may be constructed. Later in this chapter we will consider how to derive spot and forward yields from a current redemption yield curve.

C. Yield to maturity yield curve

The most commonly occurring yield curve is the yield to maturity yield curve. The equation used to calculate the yield to maturity was shown in Chapter 1. The curve itself is constructed by plotting the yield to maturity against the term to maturity for a group of bonds of the same class. Three different examples are shown at Figure 2.1. Bonds used in constructing the curve will only rarely have an exact number of whole years to redemption; however it is often common to see yields plotted against whole years on the x-axis. Figure 2.2 shows the Bloomberg page IYC for four government yield curves as at 2 December 2005; these are the US, UK, German and Italian sovereign bond yield curves.

From figure 2.2 note the yield spread differential between German and Italian bonds. Although both the bonds are denominated in euros and, according to the European Central Bank (ECB) are viewed as equivalent for collateral purposes (implying identical credit quality), the higher yield for Italian government bonds proves that the market views them as higher credit risk compared to German government bonds.

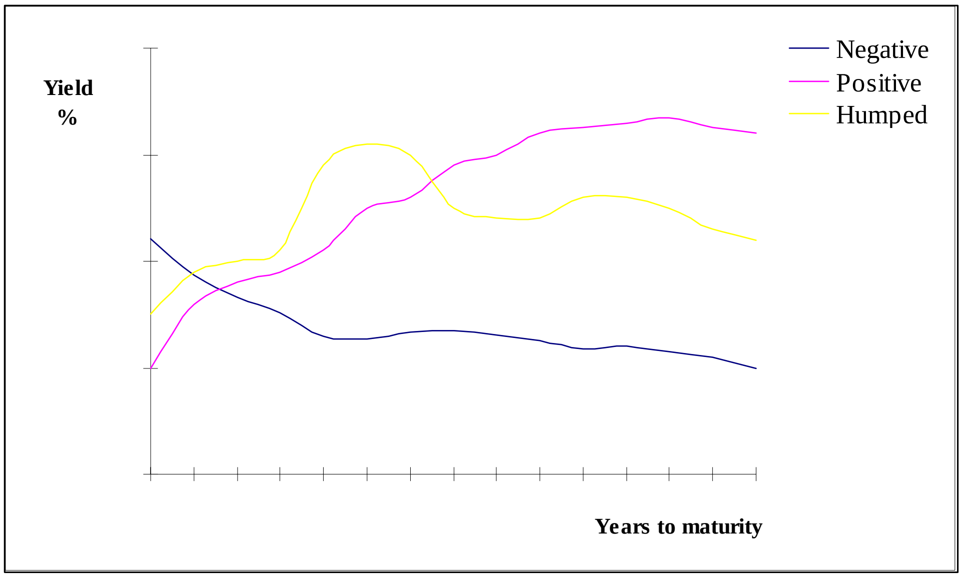

Fig 2.1 Yield to maturity yield curves

Figure 2.2 Bloomberg page IYC showing three government bond yield curves as at 2 December 2005
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The main weakness of the yield to maturity yield curve stems from the un-real world nature of the assumptions behind the yield calculation. This includes the assumption of a constant rate for coupons during the bond's life at the redemption yield level. Since market rates will fluctuate over time, it will not be possible to achieve this (a feature known as *reinvestment risk*). Only zero-coupon bondholders avoid reinvestment risk as no coupon is paid during the life of a zero-coupon bond. Nevertheless the yield to maturity curve is the most commonly encountered in markets.

For the reasons we have discussed the market often uses other types of yield curve for analysis when the yield to maturity yield curve is deemed unsuitable.

C. The par yield curve

The *par yield curve* is not usually encountered in secondary market trading, however it is often constructed for use by corporate financiers and others in the new issues or *primary* market. The par yield curve plots yield to maturity against term to maturity for current bonds trading at par. The par yield is therefore equal to the coupon rate for bonds priced at par or near to par, as the yield to maturity for bonds priced exactly at par is equal to the coupon rate. Those involved in the primary market will use a par yield curve to determine the required coupon for a new bond that is to be issued at par.

As an example consider for instance that par yields on one-year, two-year and three-year bonds are 5 per cent, 5.25 per cent and 5.75 per cent respectively. This implies that a new two-year bond would require a coupon of 5.25 per cent if it were to be issued at par; for a three-year bond with annual coupons trading at par, the following equality would be true :

$$100 = \frac{5.75}{1.0575} + \frac{5.75}{(1.0575)^2} + \frac{105.75}{(1.0575)^3}.$$

This demonstrates that the yield to maturity and the coupon are identical when a bond is priced in the market at par.

The par yield curve can be derived directly from bond yields when bonds are trading at or near par. If bonds in the market are trading substantially away from par then the resulting curve will be distorted. It is then necessary to derive it by iteration from the spot yield curve.

C. The zero-coupon (or spot) yield curve

The *zero-coupon* (or *spot*) *yield curve* plots zero-coupon yields (or spot yields) against term to maturity. In the first instance if there is a liquid zero-coupon bond market we can plot the yields from these bonds if we wish to construct this curve. However it is not necessary to have a set of zero-coupon bonds in order to construct this curve, as we can derive it from a coupon or par yield curve; in fact in many markets where no zero-coupon bonds are traded, a spot yield curve is derived from the conventional yield to maturity yield curve. This of course would be a *theoretical* zero-coupon (spot) yield curve, as opposed to the *market* spot curve that can be constructed from yields of actual zero-coupon bonds trading in the market. The zero-coupon yield curve is also known as the *term structure of interest rates*.

Spot yields must comply with equation 4.1, this equation assumes annual coupon payments and that the calculation is carried out on a coupon date so that accrued interest is zero.

$$P_d = \sum_{t=1}^{T} \frac{C}{\left(1+rs_t\right)^t} + \frac{M}{\left(1+rs_T\right)^T}$$

$$= \sum_{t=1}^{T} C \text{ x } D_t + M \text{ x } D_T \qquad (4.1)$$

where

rs_t is the spot or zero-coupon yield on a bond with t years to maturity
$D_t \equiv 1/(1 + rs_t)^t$ = the corresponding *discount factor*

In 4.1, rs_1 is the current one-year spot yield, rs_2 the current two-year spot yield, and so on. Theoretically the spot yield for a particular term to maturity is the same as the yield on a zero-coupon bond of the same maturity, which is why spot yields are also known as zero-coupon yields.

This last is an important result. Spot yields can be derived from par yields and the mathematics behind this are considered in the next section.

As with the yield to redemption yield curve the spot yield curve is commonly used in the market. It is viewed as the true term structure of interest rates because there is no reinvestment risk involved; the stated yield is equal to the actual annual return. That is, the yield on a zero-coupon bond of n years maturity is regarded as the true n-year interest rate. Because the observed government bond redemption yield curve is not considered to be the true interest rate, analysts often construct a theoretical spot yield curve. Essentially this is done by breaking down each coupon bond into a series of zero-coupon issues. For example, £100 nominal of a 10 per cent two-year bond is considered equivalent to £10 nominal of a one-year zero-coupon bond and £110 nominal of a two-year zero-coupon bond.

Let us assume that in the market there are 30 bonds all paying annual coupons. The first bond has a maturity of one year, the second bond of two years, and so on out to thirty years. We know the price of each of these bonds, and we wish to determine what the prices imply about the market's estimate of future interest rates. We naturally expect interest rates to vary over time, but that all payments being made on the same date are valued using the same rate. For the one-year bond we know its current price and the amount of the payment (comprised of one coupon payment and the redemption proceeds) we will receive at the end of the year; therefore we can calculate the interest rate for the first year : assume the one-year bond has a coupon of 10 per cent. If we invest £100 today we will receive £110 in one year's time, hence the rate of interest is apparent and is 10 per cent. For the two-year bond we use this interest rate to calculate the future value of its current price in one year's time : *this is how much we would receive if we had invested the same amount in the one-year bond.* However the two-year bond pays a coupon at the end of the first year; if we subtract this amount from the future value of the current price, the net amount is what we should be giving up in one year in return for the one remaining payment. From these numbers we can calculate the interest rate in year two.

Assume that the two-year bond pays a coupon of 8 per cent and is priced at 95.00. If the 95.00 was invested at the rate we calculated for the one-year bond (10 per cent), it would accumulate £104.50 in one year, made up of the £95 investment and coupon interest of £9.50. On the payment date in one year's time, the one-year bond matures and the two-year bond pays a coupon of 8 per cent. If everyone expected that at this time the two-year bond would be priced at more than 96.50 (which is 104.50 minus 8.00), then no investor would buy the one-year bond, since it would be more advantageous to buy the two-year bond and sell it after one year for a greater return. Similarly if the price was less than 96.50 no investor would buy the two-year bond, as it would be cheaper to buy the shorter bond and then buy the longer-dated bond with the proceeds received when the one-year bond matures. Therefore the two-year bond must be priced at exactly 96.50 in 12 months time. For this £96.50 to grow to £108.00 (the maturity proceeds from the two-year bond, comprising the redemption payment and coupon interest), the interest rate in year two must be 11.92 per cent. We can check this using the present value formula covered earlier. At these two interest rates, the two bonds are said to be in equilibrium.

This is an important result and shows that there can be no arbitrage opportunity along the yield curve; using interest rates available today the return from buying the two-year bond must equal the return from buying the one-year bond and rolling over the proceeds (or *reinvesting*) for another year. This is the known as the *breakeven principle*.

Using the price and coupon of the three-year bond we can calculate the interest rate in year three in precisely the same way. Using each of the bonds in turn, we can link together the *implied one-year rates* for each year up to the maturity of the longest-dated bond. The process is known as *boot-strapping*. The "average" of the rates over a given period is the spot yield for that term : in the example given above, the rate in year one is 10 per cent, and in year two is 11.92 per cent. An investment of £100 at these rates would grow to £123.11. This gives a total percentage increase of 23.11 per cent over two years, or 10.956% per annum (the average rate is not obtained by simply dividing 23.11 by 2, but - using our present value relationship again - by calculating the square root of "1 plus the interest rate" and then subtracting 1 from this number). Thus the one-year yield is 10 per cent and the two-year yield is 10.956 per cent.

In real-world markets it is not necessarily as straightforward as this; for instance on some dates there may be several bonds maturing, with different coupons, and on some dates there may be no bonds maturing. It is most unlikely that there will be a regular spacing of redemptions exactly one year apart. For this reason it is common for practitioners to use a software model to calculate the set of implied forward rates which best fits the market prices of the bonds that do exist in the market. For instance if there are several one-year bonds, each of their prices may imply a slightly different rate of interest. We will choose the rate which gives the smallest average price error. In practice all bonds are used to find the rate in year one, all bonds with a term longer than one year are used to calculate the rate in year two, and so on. The zero-coupon curve can also be calculated directly from the par yield curve using a method similar to that described above; in this case the bonds would be priced at par (100.00) and their coupons set to the par yield values.

The zero-coupon yield curve is ideal to use when deriving implied forward rates. It is also the best curve to use when determining the relative value, whether cheap or dear, of bonds trading in the market, and when pricing new issues, irrespective of their coupons. However it is not an accurate indicator of average market yields because most bonds are not zero-coupon bonds.

Zero-coupon curve arithmetic

Having introduced the concept of the zero-coupon curve in the previous paragraph, we can now illustrate the mathematics involved. When deriving spot yields from par yields, one views a conventional bond as being made up of an *annuity*, which is the stream of coupon payments, and a zero-coupon bond, which provides the repayment of principal. To derive the rates we can use (4.1), setting $P_d = M = 100$ and $C = rp_T$, shown below.

$$\begin{aligned} 100 &= rp_T \text{ x } \sum_{t=1}^{T} D_t + 100 \text{ x } D_T \\ &= rp_T \text{ x } A_T + 100 \text{ x } D_T \end{aligned} \qquad (4.2)$$

where rp_T is the par yield for a term to maturity of T years, where the discount factor D_T is the fair price of a zero-coupon bond with a par value of £1 and a term to maturity of T years, and where

$$A_T = \sum_{t=1}^{T} D_t = A_{T-1} + D_T \qquad (4.3)$$

is the fair price of an annuity of £1 per year for T years (with $A_0 = 0$ by convention). Substituting 4.3 into 4.2 and re-arranging will give us the expression below for the T-year discount factor.

$$D_T = \frac{1 - rp_T \text{ x } A_{T-1}}{1 + rp_T} \qquad (4.4)$$

In (4.1) we are discounting the t-year cash flow (comprising the coupon payment and/or principal repayment) by the corresponding t-year spot yield. In other words rs_t is the *time-weighted rate of return* on a t-year bond. Thus as we said in the previous section the spot yield curve is the correct method for pricing or valuing any cash flow, including an irregular cash flow, because it uses the appropriate discount factors. This contrasts with the yield-to-maturity procedure discussed earlier, which discounts all cash flows by the same yield to maturity.

4.5 The forward yield curve

The *forward* (or *forward-forward*) *yield curve* is a plot of forward rates against term to maturity. Forward rates satisfy expression (4.5) below.

$$P_d = \frac{C}{(1+{}_0rf_1)} + \frac{C}{(1+{}_0rf_1)(1+{}_1rf_2)} + \;\ldots\; + \frac{M}{(1+{}_0rf_1)\ldots\ldots(1+{}_{T-1}rf_T)}$$

$$= \sum_{t=1}^{T} \frac{C}{\prod_{i=1}^{t}(1+{}_{i-1}rf_i)} + \frac{M}{\prod_{i=1}^{T}(1+{}_{i-1}rf_i)}$$

(4.5)

where

${}_{t-1}rf_t$ is the implicit forward rate (or forward-forward rate) on a one-year bond maturing in

year t

Comparing (4.1) and (4.2) we can see that the spot yield is the *geometric mean* of the forward rates, as shown below.

$$(1+rs_t)^t = (1+{}_0rf_1)(1+{}_1rf_2)\ldots.(1+{}_{t-1}rf_t) \tag{4.6}$$

This implies the following relationship between spot and forward rates :

$$(1+{}_{t-1}rf_t) = \frac{(1+rs_t)^t}{(1+rs_{t-1})^{t-1}}$$

$$= \frac{D_{t-1}}{D_t} \tag{4.7}$$

C. Theories of the yield curve

As we can observe by analysing yield curves in different markets at any time, a yield curve can be one of four basic shapes, which are :

- **normal** : in which yields are at "average" levels and the curve slopes gently upwards as maturity increases;
- **upward sloping** (or *positive* or *rising*) : in which yields are at historically low levels, with long rates substantially greater than short rates;
- **downward sloping** (or *inverted* or *negative*) : in which yield levels are very high by historical standards, but long-term yields are significantly lower than short rates;
- **humped** : where yields are high with the curve rising to a peak in the medium-term maturity area, and then sloping downwards at longer maturities.

Various explanations have been put forward to explain the shape of the yield curve at any one time, which we can now consider.

Unbiased or pure expectations hypothesis

If short-term interest rates are expected to rise, then longer yields should be higher than shorter ones to reflect this. If this were not the case, investors would only buy the shorter-dated bonds and roll over the investment when they matured. Likewise if rates are expected to fall then longer yields should be lower than short yields. The *expectations hypothesis* states that the long-term interest rate is a geometric average of expected future short-term rates. This was in fact the theory that was used to derive the forward yield curve in (4.5) and (4.6) previously. This gives us :

$$\left(1+rs_T\right)^T=\left(1+rs_1\right)\left(1+{}_1rf_2\right)\ \ldots\ \left(1+{}_{T-1}rf_T\right) \tag{4.10}$$

or

$$\left(1+rs_T\right)^T=\left(1+rs_{T-1}\right)^{T-1}\left(1+{}_{T-1}rf_T\right) \tag{4.11}$$

where rs_T is the spot yield on a T-year bond and ${}_{t-1}rf_t$ is the implied one-year rate t years ahead. For example if the current one-year rate is rs_1 = 6.5% and the market is expecting the one-year rate in a year's time to be ${}_1rf_2$ = 7.5%, then the market is expecting a £100 investment in two one-year bonds to yield :

£100 (1.065)(1.075) = £114.49

after two years. To be equivalent to this an investment in a two-year bond has to yield the same amount, implying that the current two-year rate is rs_2 = 7%, as shown below.

$£100\,(1.07)^2 = £114.49$

This result must be so, to ensure no arbitrage opportunities exist in the market and in fact we showed as much, earlier in the chapter when we considered forward rates.

A rising yield curve is therefore explained by investors expecting short-term interest rates to rise, that is ${}_1rf_2 > rs_2$. A falling yield curve is explained by investors expecting short-term rates to be lower in the future. A humped yield curve is explained by investors expecting short-term interest rates to rise and long-term rates to fall. Expectations, or views on the future direction of the market, are a function of the expected rate of inflation. If the market expects inflationary pressures in the future, the yield curve will be positively shaped, while if inflation expectations are inclined towards disinflation, then the yield curve will be negative.

Liquidity preference theory

Intuitively we can see that longer maturity investments are more risky than shorter ones. An investor lending money for a five-year term will usually demand a higher rate of interest than if he were to lend the same customer money for a five-week term. This is because the borrower may not be able to repay the loan over the longer time period as he may for instance, have gone bankrupt in that period. For this reason longer-dated yields should be higher than short-dated yields.

We can consider this theory in terms of inflation expectations as well. Where inflation is expected to remain roughly stable over time, the market would anticipate a positive yield curve. However the expectations hypothesis cannot by itself explain this phenomenon, as under stable inflationary conditions one would expect a flat yield curve. The risk inherent in longer-dated investments, or the *liquidity preference theory*, seeks to explain a positive shaped curve. Generally borrowers prefer to borrow over as long a term as possible, while lenders will wish to lend over as short a term as possible. Therefore, as we first stated, lenders have to be compensated for lending over the longer term; this compensation is considered a premium for a loss in *liquidity* for the lender. The premium is increased the further the investor lends across the term structure, so that the longest-dated investments will, all else being equal, have the highest yield.

Segmentation Hypothesis

The capital markets are made up of a wide variety of users, each with different requirements. Certain classes of investors will prefer dealing at the short-end of the yield curve, while others will concentrate on the longer end of the market. The *segmented markets* theory suggests that activity is concentrated in certain specific areas of the market, and that there are no inter-relationships between these parts of the market; the relative amounts of funds invested in each of the maturity spectrum causes differentials in supply and demand, which results in humps in the yield curve. That is, the shape of the yield curve is determined by supply and demand for certain specific maturity investments, each of which has no reference to any other part of the curve.

For example banks and building societies concentrate a large part of their activity at the short end of the curve, as part of daily cash management (known as *asset and liability management*) and for regulatory purposes (known as *liquidity* requirements). Fund managers such as pension funds and insurance companies however are active at the long end of the market. Few institutional investors however have any preference for medium-dated bonds. This behaviour on the part of investors will lead to high prices (low yields) at both the short and long ends of the yield curve and lower prices (higher yields) in the middle of the term structure.

Further views on the yield curve

As one might expect there are other factors that affect the shape of the yield curve. For instance short-term interest rates are greatly influenced by the availability of funds in the money market. The slope of the yield curve (usually defined as the 10-year yield minus the three-month interest rates) is also a measure of the degree of tightness of government monetary policy. A low, upward sloping curve is often thought to be a sign that an environment of cheap money, due to a more loose monetary policy, is to be followed by a period of higher inflation and higher bond yields. Equally a high downward sloping curve is taken to mean that a situation of tight credit, due to more strict monetary policy, will result in falling inflation and lower bond yields. Inverted yield curves have often preceded recessions; for instance *The Economist* in an article from April 1998 remarked that in the United States every recession since 1955 bar one has been preceded by a negative yield curve. The analysis is the same: if investors expect a recession they also expect inflation to fall, so the yields on long-term bonds will fall relative to short-term bonds.

There is significant information content in the yield curve, and economists and bond analysts will consider the shape of the curve as part of their policy making and investment advice. The shape of parts of the curve, whether the short-end or long-end, as well that of the entire curve, can serve as useful predictors of future market conditions. As part of an analysis it is also worthwhile considering the yield curves across several different markets and currencies. For instance the interest-rate swap curve, and its position relative to that of the government bond yield curve, is also regularly analysed for its information content. In developed country economies the swap market is invariably as liquid as the government bond market, if not more liquid, and so it is common to see the swap curve analysed when making predictions about say, the future level of short-term interest rates.

Government policy will influence the shape and level of the yield curve, including policy on public sector borrowing, debt management and open-market operations. The markets perception of the size of public sector debt will influence bond yields; for instance an increase in the level of debt can lead to an increase in bond yields across the maturity range. Open-market operations, which refers to the daily operation by the Bank of England to control the level of the money supply (to which end the Bank purchases short-term bills and also engages in repo dealing), can have a number of effects. In the short-term it can tilt the yield curve both upwards and downwards; longer term, changes in the level of the base rate will affect yield levels. An anticipated rise in base rates can lead to a drop in prices for short-term bonds, whose yields will be expected to rise; this can lead to a temporary inverted curve. Finally debt management policy will influence the yield curve. (In the United Kingdom this is now the responsibility of the Debt Management Office.) Much government debt is rolled over as it matures, but the maturity of the replacement debt can have a significant influence on the yield curve in the form of humps in the market segment in which the debt is placed, if the debt is priced by the market at a relatively low price and hence high yield.

B. SPOT AND FORWARD RATES: Spot Rates and boot-strapping

Par, spot and forward rates have a close mathematical relationship. Here we explain and derive these different interest rates and explain their application in the markets. Note that *spot* interest rates are also called *zero-coupon* rates, because they are the interest rates that would be applicable to a zero-coupon bond. The two terms are used synonymously, however strictly speaking they are not exactly similar. Zero-coupon bonds are actual market instruments, and the yield on zero-coupon bonds can be observed in the market. A spot rate is a purely theoretical construct, and so cannot actually be observed directly. For our purposes though, we will use the terms synonymously.

A *par* yield is the yield-to-maturity on a bond that is trading at par. This means that the yield is equal to the bond's coupon level. A zero-coupon bond is a bond which has no coupons, and therefore only one cash flow, the redemption payment on maturity. It is therefore a *discount* instrument, as it is issued at a discount to par and redeemed at par. The yield on a zero-coupon bond can be viewed as a true yield, at the time that is it purchased, if the paper is held to maturity. This is because no reinvestment of coupons is involved and so there are no interim cash flows vulnerable to a change in interest rates. Zero-coupon yields are the key determinant of value in the capital markets, and they are calculated and quoted for every major currency. Zero-coupon rates can be used to value any cash flow that occurs at a future date.

Where zero-coupon bonds are traded the yield on a zero-coupon bond of a particular maturity is the zero-coupon rate for that maturity. Not all debt capital trading environments possess a liquid market in zero-coupon bonds. However it is not necessary to have zero-coupon bonds in order to calculate zero-coupon rates. It is possible to calculate zero-coupon rates from a range of market rates and prices, including coupon bond yields, interest-rate futures and currency deposits.

We illustrate shortly the close mathematical relationship between par, zero-coupon and forward rates. We also illustrate how the boot-strapping technique could be used to calculate spot and forward rates from bond redemption yields. In addition, once the discount factors are known, any of these rates can be calculated. The relationship between the three rates allows the markets to price interest-rate swap and FRA rates, as a swap rate is the weighted arithmetic average of forward rates for the term in question.

Discount Factors and the Discount Function

It is possible to determine a set of *discount factors* from market interest rates. A discount factor is a number in the range zero to one which can be used to obtain the present value of some future value. We have

$$\mathrm{PV}_t = d_t \text{ x } \mathrm{FV}_t \tag{1}$$

where

PV_t is the present value of the future cash flow occurring at time t
FV_t is the future cash flow occurring at time t
d_t is the discount factor for cash flows occurring at time t

Discount factors can be calculated most easily from zero-coupon rates; equations 2 and 3 apply to zero-coupon rates for periods up to one year and over one year respectively.

$$d_t = \frac{1}{(1 + rs_t T_t)} \tag{2}$$

$$d_t = \frac{1}{(1 + rs_t)^{T_t}} \tag{3}$$

where

d_t is the discount factor for cash flows occurring at time t
rs_t is the zero-coupon rate for the period to time t
T_t is the time from the value date to time t, expressed in years and fractions of a year

Individual zero-coupon rates allow discount factors to be calculated at specific points along the maturity term structure. As cash flows may occur at any time in the future, and not necessarily at convenient times like in three months or one year, discount factors often need to be calculated for every possible date in the future. The complete set of discount factors is called the *discount function.*

Implied Spot and Forward Rates

In this section we describe how to obtain zero-coupon and forward interest rates from the yields available from coupon bonds, using a method known as *boot-strapping*. In a government bond market such as that for US Treasuries or UK gilts, the bonds are considered to be *default-free*. The rates from a government bond yield curve describe the risk-free rates of return available in the market *today*, however they also *imply* (risk-free) rates of return for *future time periods.* These implied future rates, known as *implied forward rates*, or simply *forward rates*, can be derived from a given spot yield curve using boot-strapping. This term reflects the fact that each calculated spot rate is used to determine the next period spot rate, in successive steps.

Table 1 shows an hypothetical benchmark gilt yield curve for value as at 7 December 2000. The observed yields of the benchmark bonds that compose the curve are displayed in the last column. All rates are annualised and assume semi-annual compounding. The bonds all pay on the same coupon dates of 7 June and 7 December, and as the value date is a coupon date, there is no accrued interest on any of the bonds.[1] The *clean* and *dirty* prices for each bond are identical.

Bond	Term to maturity (years)	Coupon	Maturity date	Price	Gross Redemption Yield
4% Treasury 2001	0.5	4%	07-Jun-01	100	4%
5% Treasury 2001	1	5%	07-Dec-01	100	5%
6% Treasury 2002	1.5	6%	07-Jun-02	100	6%
7% Treasury 2002	2	7%	07-Dec-02	100	7%
8% Treasury 2003	2.5	8%	07-Jun-03	100	8%
9% Treasury 2003	3	9%	07-Dec-03	100	9%

Table 1 Hypothetical UK government bond yields as at 7 December 2000

The gross redemption yield or *yield-to-maturity* of a coupon bond describes the single rate that present-values the sum of all its future cash flows to its current price. It is essentially the *internal rate of return* of the set of cash flows that make up the bond. This yield measure suffers from a fundamental weakness in that each cash-flow is present-valued at the same rate, an unrealistic assumption in anything other than a flat yield curve environment. So the yield to maturity is an *anticipated* measure of the return that can be expected from holding the bond from purchase until maturity. In practice it will only be achieved under the following conditions:

- the bond is purchased on issue;
- all the coupons paid throughout the bond's life are re-invested at the same yield to maturity at which the bond was purchased;
- the bond is held until maturity.

[1] Benchmark gilts pay coupon on a semi-annual basis on 7 June and 7 December each year.

In practice these conditions will not be fulfilled, and so the yield to maturity of a bond is not a true interest rate for that bond's maturity period.

The bonds in table 1 pay semi-annual coupons on 7 June and 7 December and have the same time period - six months - between 7 December 2000, their valuation date and 7 June 2001, their next coupon date. However since each issue carries a different yield, the next six-month coupon payment for each bond is present-valued at a different rate. In other words, the six-month bond present-values its six-month coupon payment at its 4% yield to maturity, the one-year at 5%, and so on. Because each of these issues uses a different rate to present-value a cash flow occurring at the same future point in time, it is unclear which of the rates should be regarded as the true interest rate or benchmark rate for the six-month period from 7 December 2000 to 7 June 2001. This problem is repeated for all other maturities.

For the purposes of valuation and analysis however, we require a set of true interest rates, and so these must be derived from the redemption yields that we can observe from the benchmark bonds trading in the market. These rates we designate as rs_i, where rs_i is the *implied spot rate* or *zero-coupon rate* for the term beginning on 7 December 2000 and ending at the end of period i.

We begin calculating implied spot rates by noting that the six-month bond contains only one future cash flow, the final coupon payment and the redemption payment on maturity. This means that it is in effect trading as a zero-coupon bond, as there is only one cash flow left for this bond, is final payment. Since this cash flow's present value, future value and maturity term are known, the unique interest rate that relates these quantities can be solved using the compound interest equation (4) below.

$$\mathrm{FV} = \mathrm{PV} \times \left(1 + \frac{rs_i}{m}\right)^{(nm)}$$

$$rs_i = m \times \left(\sqrt[(nm)]{\frac{\mathrm{FV}}{\mathrm{PV}}} - 1\right) \tag{4}$$

where

FV	is the future value
PV	is the present value
rs_i	is the implied i-period spot rate
m	is the number of interest periods per year
n	is the number of years in the term

The first rate to be solved is referred to as the implied six-month spot rate and is the true interest rate for the six-month term beginning on 2 January and ending on 2 July 2000.

Equation (4) relates a cash flow's present value and future value in terms of an associated interest rate, compounding convention and time period. Of course if we re-arrange it, we may use it to solve for an implied spot rate. For the six-month bond the final cash flow on maturity is £102, comprised of the £2 coupon payment and the £100 (*par*) redemption amount. So we have for the first term, i =1, FV = £102, PV = £100, n = 0.5 years and m = 2. This allows us to calculate the spot rate as follows :

$$
\begin{aligned}
rs_i &= m \times \left(\sqrt[(nm)]{\text{FV/PV}} - 1 \right) \\
rs_1 &= 2 \times \left(\sqrt[(0.5 \text{ x } 2)]{\text{£102/£100}} - 1 \right) \\
rs_1 &= 0.04000 \\
rs_1 &= 4.000\%
\end{aligned}
\qquad (5)
$$

Thus the implied six-month spot rate or zero-coupon rate is equal to 4 per cent.[2] We now need to determine the implied one-year spot rate for the term from 7 December 2000 to 7 June 2001. We note that the one-year issue has a 5% coupon and contains two future cash flows : a £2.50 six-month coupon payment on 7 June 2001 and a £102.50 one-year coupon and principal payment on 7 December 2001. Since the first cash flow occurs on 7 June - six months from now - it must be present-valued at the 4 per cent six-month spot rate established above. Once this present value is determined, it may be subtracted from the £100 total present value (its current price) of the one-year issue to obtain the present value of the one-year coupon and cash flow. Again we then have a single cash flow with a known present value, future value and term. The rate that equates these quantities is the implied one-year spot rate. From equation (4) the present value of the six-month £2.50 coupon payment of the one-year benchmark bond, discounted at the implied six-month spot rate, is :

$$
\begin{aligned}
\text{PV}_{\text{6-mo cash flow, 1-yr bond}} &= £2.50/(1 + 0.04/2)^{(0.5 \text{ x } 2)} \\
&= £2.45098
\end{aligned}
$$

The present value of the one-year £102.50 coupon and principal payment is found by subtracting the present value of the six-month cash flow, determined above, from the total present value (current price) of the issue :

$$
\begin{aligned}
\text{PV}_{\text{1-yr cash flow, 1-yr bond}} &= £100 - £2.45098 \\
&= £97.54902
\end{aligned}
$$

The implied one-year spot rate is then determined by using the £97.54902 present value of the one-year cash flow determined above :

[2] Of course intuitively we could have concluded that the six-month spot rate was 4 per cent, without the need to apply the arithmetic, as we had already assumed that the six-month bond was a quasi-zero-coupon bond.

$$rs_2 = 2 \times \left(\sqrt[(1x2)]{£102.50/£97.54902} - 1 \right)$$
$$= 0.0501256$$
$$= 5.01256\%$$

The implied 1.5 year spot rate is solved in the same way:

$$PV_{\text{6-mo cash flow, 1.5-yr bond}} = £3.00 / (1 + 0.04 / 2)^{(0.5x2)}$$
$$= £2.94118$$

$$PV_{\text{1-yr cash flow, 1.5-yr bond}} = £3.00 / (1 + 0.0501256 / 2)^{(1x2)}$$
$$= £2.85509$$

$$PV_{\text{1.5-yr cash flow, 1.5-yr bond}} = £100 - £2.94118 - £2.85509$$
$$= £94.20373$$

$$rs_3 = 2 \times \left(\sqrt[(1.5 \text{ x } 2)]{£103 / £94.20373} - 1 \right)$$
$$= 0.0604071$$
$$= 6.04071\%$$

Extending the same process for the two-year bond, we calculate the implied two-year spot rate rs_4 to be 7.0906 per cent. The implied 2.5-year and three-year spot rates rs_5 and rs_6 are 8.1709 per cent 9.2879 per cent respectively.

The interest rates rs_1, rs_2, rs_3, rs_4, rs_5 and rs_6 describe the true zero-coupon interest rates for the six-month, one-year, 1.5-year, two-year, 2.5-year and three-year terms that begin on 7 December 2000 and end on 7 June 2001, 7 December 2001, 7 June 2002, 7 December 2002, 7 June 2003 and 7 December 2003 respectively. They are also called *implied spot rates* because they have been calculated from redemption yields observed in the market from the benchmark government bonds that were listed in table 1.

Note that the one-, 1.5-, two-year, 2.5-year and three-year implied spot rates are progressively greater than the corresponding redemption yields for these terms. This is an important result, and occurs whenever the yield curve is positively sloped. The reason for this is that the present values of a bond's shorter-dated cash flows are discounted at rates that are lower than the bond's redemption yield; this generates higher present values that, when subtracted from the current price of the bond, produce a lower present value for the final cash flow. This lower present value implies a spot rate that is greater than the issue's yield. In an inverted yield curve environment we observe the opposite result, that is implied rates that lie below the corresponding redemption yields. If the redemption yield curve is flat, the implied spot rates will be equal to the corresponding redemption yields.

Once we have calculated the spot or zero-coupon rates for the six-month, one-year, 1.5-year, two-year, 2.5-year and three-year terms, we can determine the rate of return that is implied by the yield curve for the sequence of six-month periods beginning on 7

December 2000, 7 June 2001, 7 December 2001, 7 June 2002 and 7 December 2002. These period rates are referred to as *implied forward rates* or *forward-forward rates* and we denote these as rf_i, where rf_i is the implied six-month forward interest rate today for the *i*th period.

Since the implied six-month zero-coupon rate (spot rate) describes the return for a term that coincides precisely with the first of the series of six-month periods, this rate describes the risk-free rate of return for the first six-month period. It is therefore equal to the first period spot rate. Thus we have $rf_1 = rs_1 = 4.0$ per cent, where rf_1 is the risk-free forward rate for the first six-month period beginning at period 1. We show now how the risk-free rates for the second, third, fourth, fifth and sixth six-month periods, designated rf_2, rf_3 , rf_4, rf_5 and rf_6 respectively may be solved from the implied spot rates.

The benchmark rate for the second semi-annual period rf_2 is referred to as the one-period forward six-month rate, because it goes into effect one six-month period from now ("one-period forward") and remains in effect for six months ("six-month rate"). It is therefore the six-month rate in six months time, and is also referred to as the 6-month forward-forward rate. This rate, in conjunction with the rate from the first period rf_1, must provide returns that match those generated by the implied one-year spot rate for the entire one-year term. In other words, one pound invested for six months from 7 December 2000 to 7 June 2001 at the first period's benchmark rate of 4 per cent and then reinvested for another six months from 7 June 2001 to 7 December 2001 at the second period's (as yet unknown) implied *forward* rate must enjoy the same returns as one pound invested for one year from 7 December 2000 to 7 December 2001 at the implied one-year *spot* rate of 5.0125 per cent. This reflects the law of no-arbitrage.

A moment's thought will convince us that this must be so. If this were not the case, we might observe an interest rate environment in which the return received by an investor over any given term would depend on whether an investment is made at the start period for the entire maturity term or over a succession of periods within the whole term and re-invested. If there were any discrepancies between the returns received from each approach, there would exist an unrealistic arbitrage opportunity, in which investments for a given term carrying a lower return might be sold short against the simultaneous purchase of investments for the same period carrying a higher return, thereby locking in a risk-free, cost-free profit. Therefore forward interest rates must be calculated so that they are *arbitrage-free*. Forward rates are not therefore a prediction of what spot interest rates are likely to be in the future, rather a mathematically derived set of interest rates that reflect the current spot term structure and the rules of no-arbitrage. Excellent mathematical explanations of the no-arbitrage property of interest-rate markets are contained in Ingersoll (1987), Jarrow (1996), and Robert Shiller (1990) among others.

The existence of a no-arbitrage market of course makes it straightforward to calculate forward rates; we know that the return from an investment made over a period must equal the return made from investing in a shorter period and successively reinvesting to a matching term. If we know the return over the shorter period, we are left with only one unknown, the full-period forward rate, which is then easily calculated. In our example, having established the rate for the first six-month period, the rate for the second six-month period - the one-period forward six-month rate - is determined below.

The future value of £1 invested at rf_1, the period 1 forward rate, at the end of the first six-month period is calculated as follows :

$$\begin{aligned} \text{FV}_1 &= £1 \times \left(1 + \frac{rf_1}{2}\right)^{(0.5 \text{ x } 2)} \\ &= £1 \times \left(1 + \frac{0.04}{2}\right)^1 \\ &= £1.02000 \end{aligned}$$

The future value of £1 at the end of the one-year term, invested at the implied benchmark one-year spot rate is determined as follows :

$$\begin{aligned} \text{FV}_2 &= £1 \times \left(1 + \frac{rs_2}{2}\right)^{(1x2)} \\ &= £1 \times \left(1 + \frac{0.0501256}{2}\right)^2 \\ &= £1.050754 \end{aligned}$$

The implied benchmark one-period forward rate rf_2 is the rate that equates the value of FV_1 (£1.02) on 7 June 2001 to FV_2 (£1.050754) on 7 December 2001. From equation (4) we have :

$$\begin{aligned} rf_2 &= 2 \times \left(\sqrt[(0.5 \text{ x } 2)]{\frac{\text{FV}_2}{\text{FV}_1}} - 1 \right) \\ &= 2 \times \left(\frac{£1.050754}{£1.02} - 1 \right) \\ &= 0.060302 \\ &= 6.0302\% \end{aligned}$$

In other words £1 invested from 7 December to 7 June at 4.0 per cent (the implied forward rate for the first period) and then reinvested from 7 June to 7 December 2001 at 6.0302 per cent (the implied forward rate for the second period) would accumulate the same returns as £1 invested from 7 December 2000 to 7 December 2001 at 5.01256 per cent (the implied one-year spot rate).

The rate for the third six-month period - the two-period forward six-month interest rate – may be calculated in the same way:

$FV_2 \quad = \quad £1.050754$

$$FV_3 = £1 \text{ x } (1 + rs_3 / 2)^{(1.5 \text{ x } 2)}$$
$$= £1 \text{ x } (1 + 0.0604071 / 2)^3$$
$$= £1.093375$$

$$rf_3 = 2 \times \left(\sqrt[(0.5\times 2)]{\frac{FV_3}{FV_4}} - 1 \right)$$
$$= 2 \times \left(\sqrt[1]{£1.093375 / £1.050754} - 1 \right)$$
$$= 0.081125$$
$$= 8.1125\%$$

In the same way the three-period forward six-month rate rf_4 is calculated to be 10.27247 per cent. The rest of the results are shown in table 2. We say *one-period* forward rate because it is the forward rate that applies to the six-month period. The results of the implied spot (zero-coupon) and forward rate calculations along with the given redemption yield curve are illustrated graphically in Figure 1.

The simple bootstrapping methodology can be applied using a spreadsheet for actual market redemption yields. However in practice we will not have a set of bonds with exact and/or equal periods to maturity and coupons falling on the same date. Nor will they all be priced conveniently at par. In designing a spreadsheet spot rate calculator therefore, the coupon rate and maturity date is entered as standing data and usually interpolation is used when calculating the spot rates for bonds with uneven maturity dates. A spot curve model that uses this approach in conjunction with the boot-strapping method is available for downloading at www.yieldcurve.com Market practitioners usually use discount factors to extract spot and forward rates from market prices. For an account of this method, see Choudhry *et al* (2001), chapter 9.

Term to maturity	**Yield to maturity**	**Implied spot rate**	**Implied one-period forward rate**
0.5	4.0000%	4.00000%	4.00000%
1	5.0000%	5.01256%	6.03023%
1.5	6.0000%	6.04071%	8.11251%
2	7.0000%	7.09062%	10.27247%
2.5	8.0000%	8.17090%	12.24833%
3	9.0000%	9.28792%	14.55654%

Table 2 Implied spot and forward rates

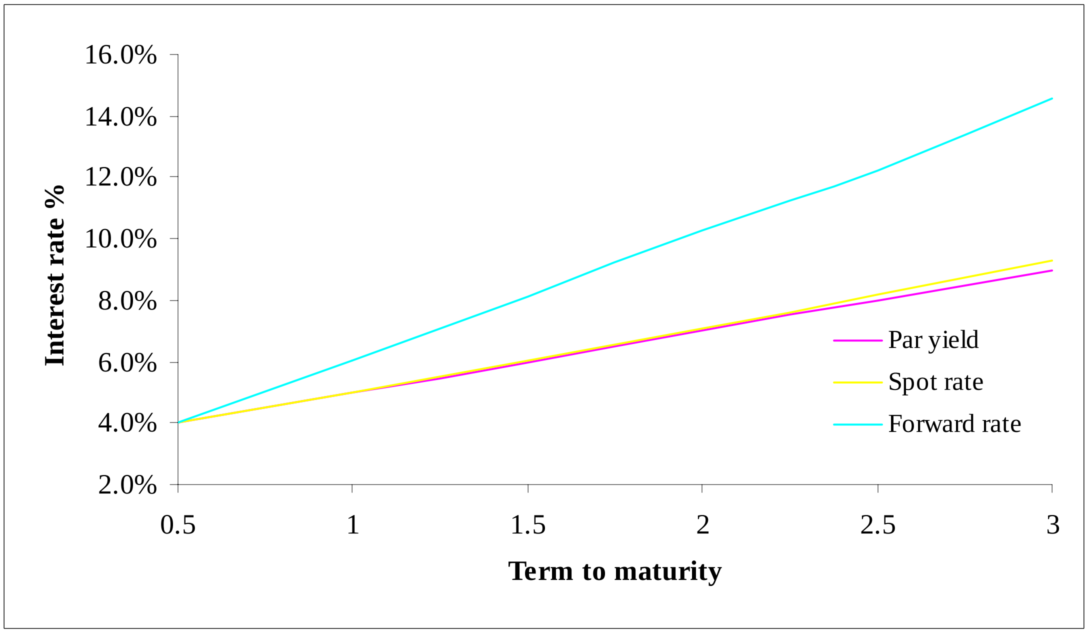

Figure 1 Par, spot and forward yield curves

Examples

Example 1
Consider the following spot yields :

1-year	10%
2-year	12%

Assume that a bank's client wishes to lock in *today* the cost of borrowing 1-year funds in one year's time. The solution for the bank (and the mechanism to enable the bank to quote a price to the client) involves raising 1-year funds at 10% and investing the proceeds for two years at 12%. As we observed above, the no-arbitrage principle means that the same return must be generated from both fixed rate and reinvestment strategies.

Using the following formula:

$$(1 + rs_2)^2 = (1 + rs_1)(1 + rf)$$

$$rf = \frac{(1 + y_2)^2}{(1 + y_1)} - 1$$

the relevant forward rate is calculated to be 14.04 per cent.

Example 2

If a 1-year AAA Eurobond trading at par yields 10% and a 2-year Eurobond of similar credit quality, also trading at par, yields 8.75%, what should be the price of a 2-year AAA zero-coupon bond? Note that Eurobonds pay coupon annually.

(a)	Cost of 2-year bond (per cent nominal)	100
(b)	*less* amount receivable from sale of first coupon on this bond (that is, its present value)	= 8.75 / 1 + 0.10 = 7.95
(c)	*equals* amount that must be received on sale of second coupon plus principal in order to break even	92.05
(d)	calculate the yield implied in the cash flows below (that is, the 2-year zero-coupon yield) - receive 92.05 - pay out on maturity 108.75	
	Therefore Gives R equal to 8.69%	$92.05 = 108.75 / (1 + R)^2$
(e)	What is the price of a 2-year zero-coupon bond with nominal value 100, to yield 8.69%?	= (92.05 / 108.75) x 100 = 84.64

Example 3

A highly-rated customer asks you to fix a yield at which he could issue a 2-year zero-coupon USD Eurobond in three years' time. At this time the US Treasury zero-coupon rates were :

1 Yr	6.25%
2 Yr	6.75%
3 Yr	7.00%
4 Yr	7.125%
5 Yr	7.25%

(a) Ignoring borrowing spreads over these benchmark yields, as a market maker you could cover the exposure created by borrowing funds for 5 years on a zero-coupon basis and placing these funds in the market for 3 years before lending them on to your client. Assume annual interest compounding (even if none is actually paid out during the life of the loans)

Borrowing rate for 5 years $\left[\frac{R_5}{100}\right] = 0.0725$

Lending rate for 3 years $\left[\frac{R_3}{100}\right] = 0.0700$

(b) The key arbitrage relationship is :

Total cost of funding = Total Return on Investments

$$(1+R_5)^5 = (1+R_3)^3 \text{ x } (1+R_{3x5})^2$$

Therefore the break-even forward yield is :

$$R_{3x5} = \sqrt[2]{\left[\frac{(1+R_5)^5}{(1+R_3)^3}\right]} - 1$$

$$= \quad 7.63\%$$

Example 4
Forward rate calculation for money market term

Consider two positions:
repo of £100 million gilts GC from 2 January 2000 for 30 days at 6.500%,
reverse repo of £100 million gilts GC from 2 January for 60 days at 6.625%.

The two positions can be said to be a 30-day forward 30-day (repo) interest rate exposure (a 30 versus 60 day forward rate). What forward rate must be used if the trader wished to hedge this exposure, assuming no bid-offer spreads and a 360-day base?

The 30-day by 60-day forward rate can be calculated using the following formula :

$$rf = \left[\left(\frac{1+\left(rs_2 \times \frac{L}{M}\right)}{1+\left(rs_1 \times \frac{S}{M}\right)}\right) - 1\right] \times \frac{M}{L-S}$$

where

rf	is the forward rate
rs_2	is the long period rate
rs_1	is the short period rate
L	is the long period days
S	is the short period days
M	is the day-count base

Using this formula we obtain a 30 v 60 day forward rate of 6.713560%.

This interest rate exposure can be hedged using interest rate futures or Forward Rate Agreements (FRAs). Either method is an effective hedging mechanism, although the trader must be aware of :

- *basis* risk that exists between Repo rates and the forward rates implied by futures and FRAs;
- date mismatched between expiry of futures contracts and the maturity dates of the repo transactions.

Forward Rates and Compounding

Examples 1-3 above are for forward rate calculations more than one year into the future, and therefore the formula used must take compounding of interest into consideration. Example 4 is for a forward rate within the next 12 months, with one-period bullet interest payments. A different formula is required to account for the sub-one year periods, as shown in the example.

C. Understanding Forward Rates

Spot and forward rates that are calculated from current market rates follow mathematical principles to establish what the market believes the arbitrage-free rates for dealing *today* at rates that are effective at some point in the future. As such forward rates are a type of market view on where interest rates will be (or should be!) in the future. However forward rates are not a prediction of future rates. It is important to be aware of this distinction. If we were to plot the forward rate curve for the term structure in three months time, and then compare it in three months with the actual term structure prevailing at the time, the curves would almost certainly not match. However this has no bearing on our earlier statement, that forward rates are the market's expectation of future rates. The main point to bear in mind is that we are not comparing like-for-like when plotting forward rates against actual current rates at a future date. When we calculate forward rates, we use the current term structure. The current term structure incorporates all known information, both economic and political, and reflects the market's views. This is exactly the same as when we say that a company's share price reflects all that is known about the company and all that is expected to happen with regard to the company in the near future, including expected future earnings. The term structure of interest rates reflects everything the market knows about relevant domestic and international factors. It is this information then, that goes into the forward rates calculation. In three months time though, there will be new developments that will alter the market's view and therefore alter the current term structure; these developments and events were (by definition, as we cannot know what lies in the future!) not known at the time we calculated and used the three-month forward rates. This is why rates actually turn out to be different from what the term structure predicted at an earlier date. However for dealing today we use today's forward rates, which reflect everything we know about the market today.

B. THE TERM STRUCTURE OF INTEREST RATES

We illustrate a more advanced description of what we have just discussed. It is used to obtain a zero-coupon curve, in the same way as seen previously, but just using more formal mathematics.

Under the following conditions:

- frictionless trading conditions;
- competitive economy;
- discrete time economy;

with discrete trading dates of $\{0,1,2,....., \tau\}$, we assume a set of zero-coupon bonds with maturities $\{0,1,2,....., \tau\}$. The price of a zero-coupon bond at time *t* with a nominal value of £1 on maturity at time *T* (such that $T \geq t$) is denoted with the term *P*(*t*, *T*). The bonds are considered risk-free.

The price of a bond at time *t* of a bond of maturity *T* is given by

$$P(t,T) = \frac{1}{[y(t,T)]^{(T-t)}}$$

where *y*(*t*, *T*) is the yield of a *T*-maturity bond at time *t*. Re-arranging the above expression, the yield at time *t* of a bond of maturity *T* is given by

$$y(t,T) = \left[\frac{1}{P(t,T)}\right]^{1/(T-t)}.$$

The time *t* forward rate that applies to the period [*T*, *T*+1] is denoted with *f*(*t*, *T*) and is given in terms of the bond prices by

$$f(t,T) = \frac{P(t,T)}{P(t,T+1)}.$$

This forward rate is the rate that would be charged at time *t* for a loan that ran over the period [*T*, *T*+1].

From the above expression we can derive an expression for the price of a bond in terms of the forward rates that run from *t* to *T*-1, which is

$$P(t,T) = \frac{1}{\prod_{j=t}^{T-1} f(t,j)}.$$

This expression means:

$\prod_{j=t}^{T-1} f(t, j) = f(t,t) \cdot f(t,t+1) \ldots f(t,T-1)$, that is, the result of multiplying the rates that apply to the interest periods in index *j* that run from *t* to *T*-1. It means that the price of a bond is equal to £1 received at time *T*, that has been discounted by the forward rates that apply to the maturity periods up to time *T*-1.

The expression is derived as shown below:

Consider the following expression for the forward rate applicable to the period (*t*, *t*),

$$f(t,t) = \frac{P(t,t)}{P(t,t+1)}$$

but of course *P*(*t*, *t*) is equal to 1, so therefore

$$f(t,t) = \frac{1}{P(t,t+1)}$$

which can be re-arranged to give

$$P(t,t+1) = \frac{1}{f(t,t)}.$$

For the next interest period we can set

$$f(t,t+1) = \frac{P(t,t+1)}{P(t,t+2)}$$

which can be re-arranged to give

$$P(t,t+2) = \frac{P(t,t+1)}{f(t,t+1)}.$$

We can substitute the expression for *f*(*t*, *t*+1) into the above and simplify to give us

$$P(t,t+2) = \frac{1}{f(t,t)f(t,t+1)}.$$

If we then continue for subsequent interest periods (*t*, *t*+3) onwards, we obtain

$$P(t,t+j) = \frac{1}{f(t,t)f(t,t+1)f(t,t+2)\ldots f(t,t+j-1)}$$

which is simplified into our result above.

Given a set of risk-free zero-coupon bond prices, we can calculate the forward rate applicable to a specified period of time that matures up to the point T-1. Alternatively, given the set of forward rates we are able to calculate bond prices.

The zero-coupon or spot rate is defined as the rate applicable at time t on a one-period risk-free loan (such as a one-period zero-coupon bond priced at time t). If the spot rate is defined by $r(t)$ we can state that

$$r(t) = f(t, t).$$

This spot rate is in fact the return generated by the shortest-maturity bond, shown by

$$r(t) = y(t, t+1).$$

We can define forward rates in terms of bond prices, spot rates and spot rate discount factors.

The box below shows bond prices for zero-coupon bonds of maturity value $1. We can plot a yield curve based on these prices, and we see that we have obtained forward rates based on these bond prices, using the technique described above.

Example
Zero-coupon bond prices, spot rates and forward rates

Period	Bond price [*P(0, T)*]	Spot rates	Forward rates
0	1		
1	0.984225	1.016027	1.016027
2	0.967831	1.016483	1.016939
3	0.951187	1.016821	1.017498
4	0.934518	1.017075	1.017836
5	0.917901	1.017280	1.018102
6	0.901395	1.017452	1.018312
7	0.885052	1.017597	1.018465
8	0.868939	1.017715	1.018542
9	0.852514	1.017887	1.019267

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