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The Synthetic Collateralised Debt Obligation: analysing the Super-Senior Swap element

Nicoletta Baldini *

July 2003

Basic Facts

In a typical cash flow securitization a SPV (Special Purpose Vehicle) transfers interest income and principal repayments from a portfolio of risky assets, the so called asset pool, to a prioritized set of tranches. The level of credit exposure of every single tranche depends upon its level of subordination: so, the junior tranche will be the first to bear the effect of a credit deterioration of the asset pool, and senior tranches the last.

The asset pool can be made up by either any type of debt instrument, mainly bonds or bank loans, or Credit Default Swaps (CDS) in which the SPV sells protection[1]. When the asset pool is made up solely of CDS contracts we talk of 'synthetic' Collateralized Debt Obligations (CDOs); in the so called 'semi-synthetic' CDOs, instead, the asset pool is made up by both debt instruments and CDS contracts. The tranches backed by the asset pool can be funded or not, depending upon the fact that the final investor purchases a true debt instrument (note) or a mere synthetic credit exposure.

Generally, when the asset pool is constituted by debt instruments, the SPV issues notes (usually divided in more tranches) which are sold to the final investor; in synthetic CDOs, instead, tranches are represented by basket CDSs with which the final investor sells protection to the SPV. In any case all the tranches can be interpreted as percentile basket credit derivatives and their degree of subordination determines the percentiles of the asset pool loss distribution concerning them

It is not unusual to find both funded and unfunded tranches within the same securitisation: this is the case for synthetic CDOs (but the same could occur with semi-synthetic CDOs) in which notes are issued and the raised cash is invested in risk free bonds that serve as collateral.

Super Senior Position

Our analysis will now focus upon the so called 'Super Senior position'. By *Super Senior* position we mean a contract whose seniority is even 'higher' than that of a senior tranche (rated AAA): that is, the SPV first carries out its payments to the super senior counterpart and only after such payments are made, it starts the allocation of flows to the different tranches according to the "waterfall". Super Senior contracts are typically associated with synthetic or semi-synthetic CDOs, in which the vehicle buys from a counterpart protection on the Super Senior tranche via a basket CDS.

[1] In a CDS, the protection seller pays to the protection buyer a certain amount of money (generally equal to the loss given default, which is the nominal amount less the recoveries) in case the name underlying the CDS contract is hit by a 'credit event' before a specified maturity date.

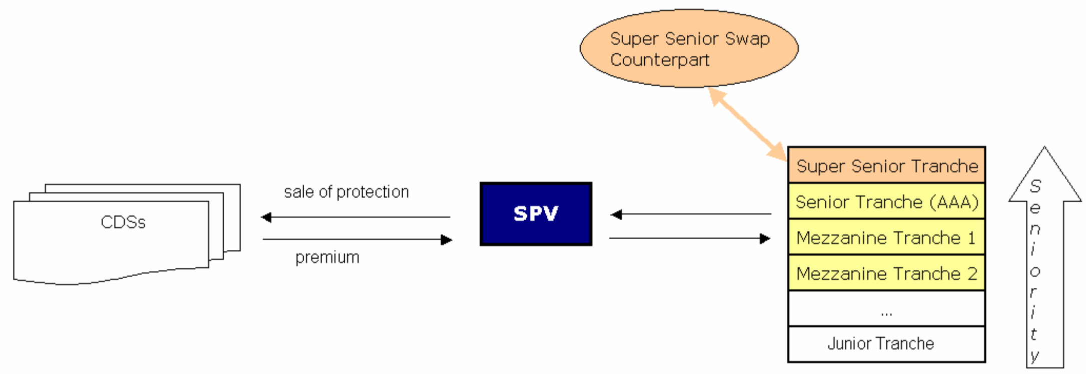

Figure1 – Synthetic CDO Structure (Unfunded)

Why is this position so important? The answer is suggested in the following statement by Jeff Huffman, a London-based executive director at Goldman Sachs' credit derivatives group: "*The future growth in managed synthetic collateralised debt obligation (CDO) deal-flow is dependent on the willingness of counterparties to sell super-senior risk protection to CDOs*".[2] Generally, we can say that a Super Senior position is exposed to the tail of the loss distribution and, in particular, to the joint extreme events of default among more names and/or between credit spreads and market interest rates.

For example, let's suppose to have a so-called *repackaged securities* transaction where the asset pool is made up by the sale of protection upon several single name CDSs; let us also suppose that the SPV issues fixed rate notes (no tranching for simplicity's sake), buys collateral and that the SPV also enters into a swap contract with the same CDS counterpart (S_ctp hereafter) in order to neutralize the risks arising from the mismatch between inflows and outflows: in such a swap the SPV pays to S_ctp the flows received from the collateral and from the premia on the sold CDSs, receiving in turn a coupon which will make up the rate paid to the investor in the issued notes. In case of a credit event concerning one of the reference entities underlying the CDSs, part of the collateral equal to the sum of the due credit payments and the unwinding of the corresponding portion of the asset swap will be cash settled.

As an extreme example, in case of default of (all of) the reference entities, the collateral might not be worth enough to guarantee contingent credits in favour of S_ctp. The risk profile taken by S_ctp is strictly dependent upon the type of collateral, the reference entities underlying the CDSs and the structure of the flows from the swap contract and it is called Super Senior risk because, in any case, the SPV first meets its payments to S_ctp and only after that pays the note holders.

[2]Risk Magazine, November 2002, Vol. 15, N.11

How can we quantify and monitor such risks? It is possible, using Monte Carlo scenarios that simulate the times of default and the evolution of the relevant market variables, to obtain the loss distribution function of the asset pool. The analysis of such a distribution provides information upon the magnitude and type of the losses: its average is the expected loss, its second moment suggests its variability, whereas worst case estimates for different confidence levels can be inferred from the corresponding percentiles.

Such indicators can be monitored during the whole life of the transaction through the risk factors underpinning its evolution:

1. recovery rate
2. credit spread
3. term structure
4. volatility of the above elements

Modelling issues

How can we estimate the loss distribution that correctly takes into account both credit and market risk and that, at the same time, accounts for their joint dynamics? The joint loss distribution depends upon the single loss distributions of each name as well as on their correlation. Therefore, we need to estimate the loss function for each name and specify a model that ties them up. In literature there are several models that specify the loss function of a reference entity. Probably, the most widespread approach to the risk of default is based upon the definition of instantaneous risk of default.[3]

The default can be described by a *survival function*:

$S(t)=\Pr\{T>t\}$

which indicates the probability that a security will attain time t. The survival time T is called *time until default*. This is the same as to say that the probability distribution function of the survival time T can be specified by $F(t)=\Pr\{T<t\}$ which gives the probability that default occurs before t. The corresponding probability density function is

$$f(t) = \frac{dF(t)}{dt}$$

The hazard rate function indicates the entity's default probability over the time interval $[x, x+\Delta t]$ if it has survived to time x:

$$\Pr\{x < T \leq x + \Delta t \mid T > x\} = \frac{F(x+\Delta t) - F(x)}{1 - F(x)} \cong \frac{f(x)\Delta t}{1 - F(x)}$$

while the hazard rate function is

$$\lambda(x) = \frac{f(x)}{1 - F(x)} = -\frac{S'(x)}{S(x)}.$$

[3] See Li (2000) and Duffie (1998)

The survival function can be expressed in terms of the hazard rate function

$$S(t) = e^{-\int_0^t \lambda(u)du}.$$

A typical assumption is that the hazard rate is a constant. In this case the survival time follows an exponential distribution with parameter λ:

$$S(t) = e^{-\lambda \cdot t}.$$

The idea behind the Duffie approach is that the credit event can be (approximately)[4] modelled as a Poisson process, with intensity rate (or hazard rate) λ depending on the length of the time interval. This implies that the probability of observing a credit event between time 0 and time *t* is equal to λ **t*.

For each name we can estimate the intensity of default (λ) by using the market quotes for CDSs[5]. In particular, we must solve the basic financial equivalence concerning CDS, according to which the 'buy protection' leg and the 'sell protection' leg must be equal (at inception a CDS is worth 0).

If $p^i(T_m)$ is the CDS premium on credit *i*-th with maturity T_m, assuming a deterministic recovery rate (RR^i) and no dependence between risk free rates and defaults we have:

$$p^i(T_m) \cdot \sum_{k=1}^{m} \tau_k \cdot df\_RiskFree(T_k) \cdot CNDP^i(T_k) = (1 - RR^i) \cdot \int_0^{T_m} df\_RiskFree(u) \cdot DP(du)$$

The first term in the equation is the present value of the payments of the spread $p^i(T_m)$ which is paid at each T_k provided there has been no default yet (τ_k is the day-count fraction for the period k); the integral, instead, describes the present value of the payment of (1-RR^i) at the time of default.

Assuming an *a priori* hypothesis on the recovery rate (on that maturity and for that CDS), the only unknown in the preceding equation is the very intensity of default used to calculate CNDPs (Cumulative No Default Probability) and DP (Default Probability)[6].

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[4] The values that a Poisson random variable can take are all the positive integers, whereas we are interested only in two events: the firm defaults (0) or doesn't default (1). The general form of the Poisson distribution is, for x=0,1,2, …:

$$Poisson(x;\lambda) = \frac{e^{-\lambda \cdot t}(\lambda \cdot t)^x}{x!}$$

For a single firm, we are interested only in the case x=0 , i.e. the firm does not default, whereas for x>0, the firm defaults.

[5] The hazard rate function can be obtained from historical default rates provided by rating agencies, by using the Merton approach or estimating it through market observable information (CDS, asset swap, risky corporate bond prices).

[6] In case we wanted to estimate the intensity of default function using more CDS quotes for the same name but referring to different maturities, in order for the system of financial equivalences o have a solution, we must specify either a function piecewise constant or with linear interpolation.

CDOs are mainly correlation products and, as such, their correct pricing needs a model that can account for both joint credit events and their link to the interest rate component. When we talk of 'joint credit events' we are not necessarily referring to catastrophic events that simultaneously affect several names (i.e. economic shocks such as liquidity breakdowns or sovereign risks) but also, more simply, to a common macroeconomic scenario that may influence the credit standing of firms belonging to the same industry (even if with different intensities) or specific relations among subsets of names tied together by contractual agreements or capital structure.

The intensity of default for each name (that is, the conditional expected arrival rate of default) can in turn depend upon other variables, observable or not. The correlation effect among events of default can be introduced in a framework, where the default of each single entity depends upon a common factor, not only upon the specific state of the entity itself[7]. Therefore, the intensity of default estimated in the way showed above can be interpreted as the sum of two components, a common and a specific one.

For the common factor, we assume a Poisson distribution with intensity Λ; we can think to specify the dependence between the intensities of default of each name and the common factor as

$$\hat{\lambda}_i = l_i \Lambda + \lambda_i$$

where l_i represents the probability that the i-th entity defaults in case of a common credit event and λ_i is the intensity of arrival of default specific to entity i. The parameters of the process are estimated by calibrating the prices produced by the model on the credit spreads given by the market and their significance is tested by econometric analysis. More in detail, we could apply factorial techniques[8] to the analysis of the intensities taken from market data, in such a way to estimate, under certain conditions, both the contribution of the two components (common and specific) and the weighs l_i.

Under the hypothesis of independence of all the Poisson processes involved, the arrival time T of a credit event is distributed according to an exponential distribution with parameter

$$\mathrm{H} = \Lambda + \sum_i \lambda_i .$$

Based upon the process explained above, let us simulate the relevant times. The arrival times of credit events are generated according to the methodology known as 'recursive event time algorithm'.[9]

[7] See Duffie- Singleton (1999)
[8] See Joreskog K.G. – D. Sorbom (1979)
[9] See Duffie-Singleton (1999)

More specifically:

1. given the simulated history up to the last relevant time T_k, we simulate the next T_{k+1}; if T_{k+1} is greater than the maturity date, we come to an end
2. otherwise, we simulate whether the credit event concerns the common factor or a specific entity:
   - credit event for the common factor with probability $\frac{\Lambda}{H}$: therefore we simulate the entities affected by contagion (each con with probability l_i)
   - credit event for the specific entity (with probability 1-$\frac{\Lambda}{H}$): we simulate which one among them has defaulted (the i-th entity has probability $q_i = \frac{\lambda_i}{\sum \lambda_i}$)

   we update the state of the system, removing all the entities defaulted, in order to generate the next relevant time:
3. we substitute k with k+1 and go back to point 1.

We want to emphasize that the above illustrated model allows us to include extreme scenarios, such as:

- default of both all the reference entities and collateral: the relevant date is that of the common factor and the contagion spreads out for all underlying entities;
- no default: the relevant date is that of the common factor but no underlying is affected by contagion

The common factor may be an observable or an unobservable variable. A desirable result would be to estimate the weighs l_i in such a way to suggest the interpretation of the common factor as a macroeconomic variable[10]: an appropriate choice might be the short rate driver for the interest rate term structure. Bearing this idea in mind, let's specify the stochastic model for the evolution of the short rate: a common choice is an extended Vasicek type model. At this point the relationship between the intensity Λ of the common factor and the short rate r_t can be thought as linear, namely

$$\Lambda_t = max[\alpha + \beta \cdot r_t, 0].$$

This relationship allows us to imagine different credit-market correlation scenarios corresponding to appropriate choices of α and β. Proceeding backwards along the *recursive event time algorithm*, we reckon the level of the short rate r_{T_k} on every relevant date T_k and consequently update the intensity of the common factor.

[10] This can be achieved using techniques of factorial rotation; a good reference is Lewis-Beck (1994)

Conclusions

The Super Senior position is at the very top of the capital structure, so writing protection on it is equivalent to supplying the CDO with catastrophic risk protection. The model for a correct pricing of the Super Senior component cannot leave aside the need for a deep understanding of the effects of correlation among credit events and between these and macroeconomic variables. Since a Super Senior tranche is a typical correlation product, it turns out that it is necessary to stress this component.

In order to do that, factorial models are often used: they allow to estimate linear relationships between the intensities of default of different names (to calibrate using market data) and those of macro variables (observable or not). We can therefore reckon tail events probability and, more generally, the complete loss distribution.

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Nicoletta Baldini is a quantitative analyst at MPS Finance Banca Mobiliare SpA in Siena, Italy.

nicoletta.baldini@mpsfinance.it