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Learning Curve
A Primer on Convertible Bonds

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Introduction

The reasons for using equity-linked structured products are many and varied. Corporates may be motivated by the need to raise capital at a competitive rate of interest, to fund a merger or acquisition or to manage their balance sheets. Whatever an individual company's reason for using such instruments, cost of capital, capital management and tax considerations are usually of prime importance. These days though even individual investors are using these products as tools that can help to attain their investment goals. In times when equity markets are bullish this might be to forego interest on a deposit account in order to participate in stock market growth. When stock markets fall, investors' funds may be divided between a bond paying a high coupon and equity index participation with a guaranteed return of investment. The paramount goals for the individual investor will be to obtain a higher rate of interest than that available from their local building society/mortgage or bank deposit account, or to participate in rises in a stock market index or maybe both.

This Learning Curve focuses on the convertible bond. These instruments have moved on considerably from their original function, namely exchange of a debt instrument for equity, and can possess features which make them difficult to price and hedge effectively. This complexity, however, can in turn lead to potential arbitrage opportunities for other market players to exploit.

Convertible bonds

Convertible bonds (CBs) represent an exciting but rather neglected section of the fixed income markets. This type of bond endows on its holder the right to purchase a defined quantity of shares at a defined price. This is achieved by returning the bond to the issuer on maturity (or in some cases earlier), and receiving in return the specified amount of equity. In the absence of a company's bankruptcy or default there will be a floor value to the convertible bond, for example, its redemption value at maturity. In the event that the price of the company's shares rises, the value of the bond will also rise but not necessarily in a perfectly correlated way. Expressed simply the bond has features of both fixed income and equity instruments and these features combined create a profile very much akin to that of a call option. This profile is identified in Figure 1, which illustrates the traded profile for a Deutsche Bank CB and in Figure 3, which presents a theoretical payout structure of a CB at expiration. The basics of CBs will be covered in more detail later.

The hybrid equity/fixed income features of CBs make them attractive to a wide cross-section of market participants. At one end of the scale are risk-averse, fixed-income portfolio managers looking for returns linked to equity exposure while at the other end of the scale are risk-loving investors speculating that distressed paper will recover and, in the process of recovery, deliver a high return.

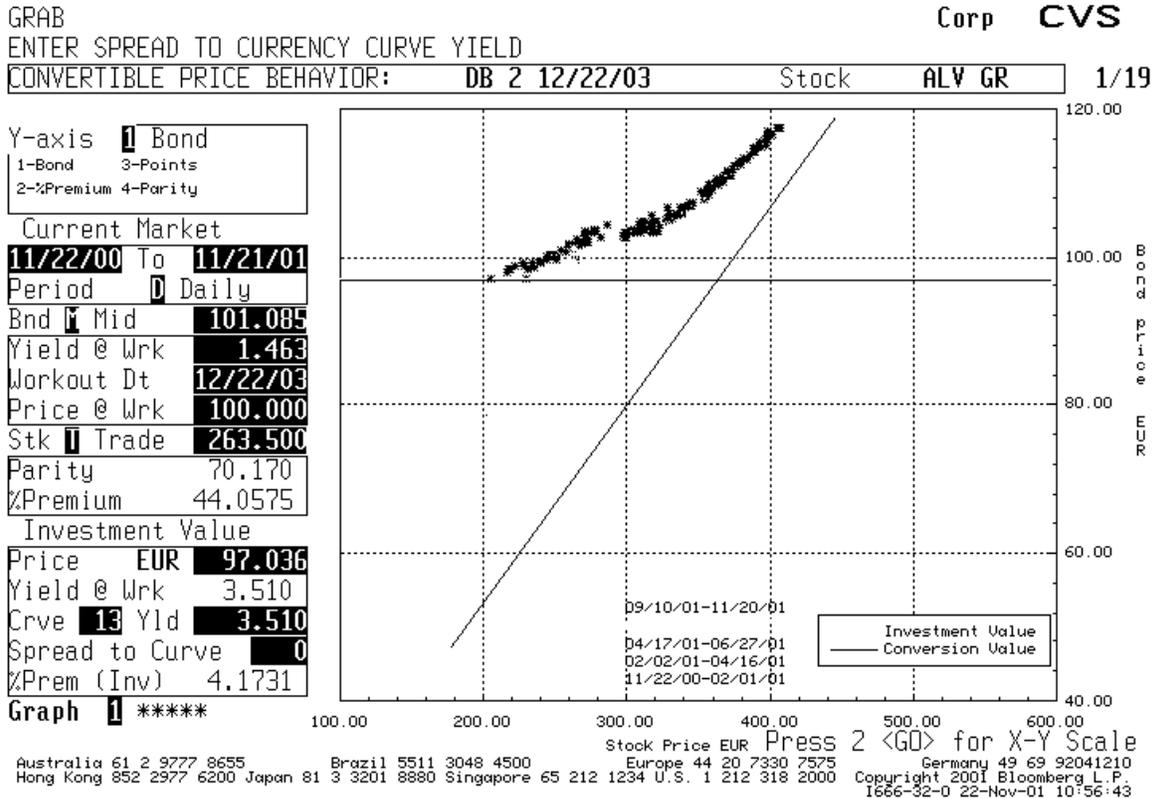


Figure 1: The traded profile for a Deutsche Bank convertible bond.
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Basics of convertible bonds

Even in their most basic form convertible bonds can be difficult to analyse and hedge. Nevertheless they offer a wide cross-section of market participants the opportunity to invest indirectly in the shares of a company with limited downside risk. Before discussing these instruments in detail it is essential to get to grips with some market jargon and standard descriptive statistics that are used to describe and analyse CBs. To help in the process it is useful to consider the terms describing a typical convertible bond. Table 1 presents the specification, or indenture, of such a bond. Expressions such as coupon, maturity and par value will be familiar and need no further explanation. The issue size of £50,000,000 is self-explanatory and is broken down into 50,000 bonds each with a par value of £1000. Other terms such as conversion price, conversion ratio, parity and premium need explanation.

Conversion price refers to the price that the holder of the bond will need to pay when converting the bond into shares. In this illustrative example the conversion price is fixed at £12.50. The number of shares that the holder of the bond can buy at the specified conversion price is fixed by the conversion ratio, here 80.

Table 1

Coupon (annual)	3.00%
Maturity	26 January 2012
Issue size	£50,000,000
Denomination or par value	£1000
Number of bonds	50,000
Issue price	100%
Conversion price	£12.50
Conversion ratio (shares/bond)	80
Current share price	£8.50
Parity	68%
Conversion premium	53.85%

As an equation the conversion ratio is defined as:

$$\text{Conversion ratio} = \frac{\text{Denomination}}{\text{Conversion price}} \quad (1)$$

$$\text{Conversion ratio} = \frac{£1000}{£12.50} = 80. \quad (2)$$

Parity is an important concept in the analysis and understanding of convertible bonds. In a sense parity reflects the in-, at-, or out-of-the-moneyness of the bond at currently prevailing share prices in the market. In the case of a single currency bond, parity can be defined as:

$$\text{Parity} = \text{Share price} \times \text{Conversion ratio}$$

but is normally referred to as a percentage of the CB's par value:

$$\text{Parity} = \frac{(\text{Share price}) \times (\text{Conversion ratio})}{\text{Bond Par Value}} \times 100. \quad (3)$$

Thus if the conversion ratio is set at 80 (Denomination/Conversion price) and the market price of the stock at the bond's maturity date is £8.50, parity, expressed as a percentage, will be 68%.

$$\text{Parity} = \left[\frac{(\text{£}8.50) \times (80)}{\text{£}1000} \right] \times 100 = 68\%. \quad (3')$$

As mentioned above, in practice the value found using equation (3') is expressed as a percentage, so that parity would be quoted as 68% and referred to as 68 bond points. At this share price a rational investor will not wish to convert the bond into shares. The bond has a redemption value of £1000 while the market price of the shares is only £680. Viewed from an equity perspective the bond is out-of-the-money with a current parity of 68 bond points. Repeated application of equation (12.3') for a continuum of stock prices will generate a parity line. This line appears as depicted in Figure 12.2. The parity line itself indicates the total current market value of the shares associated with the bond were it to be converted at that share price.

Continuing the examination of a CB from the equity point of view, there is evidently a premium involved in gaining exposure to the company's equity through the purchase of the bond rather than a direct purchase of the company's equity. Using the same example as above an intuitively appealing calculation would be to find the difference between the value of the equity through conversion (£1000) and current parity (£680) yielding a premium of £320. Alternatively premium can be calculated as the difference between the CB's price and parity multiplied by 100 divided by parity. The resulting statistic defines, in percentage terms, the excess that an investor needs to pay in order to gain exposure to the number of shares specified in the conversion ratio. A more formal representation of this statistic is presented in equation (4) while equation (4') calculates 47.06% as the premium attached to the CB under examination.

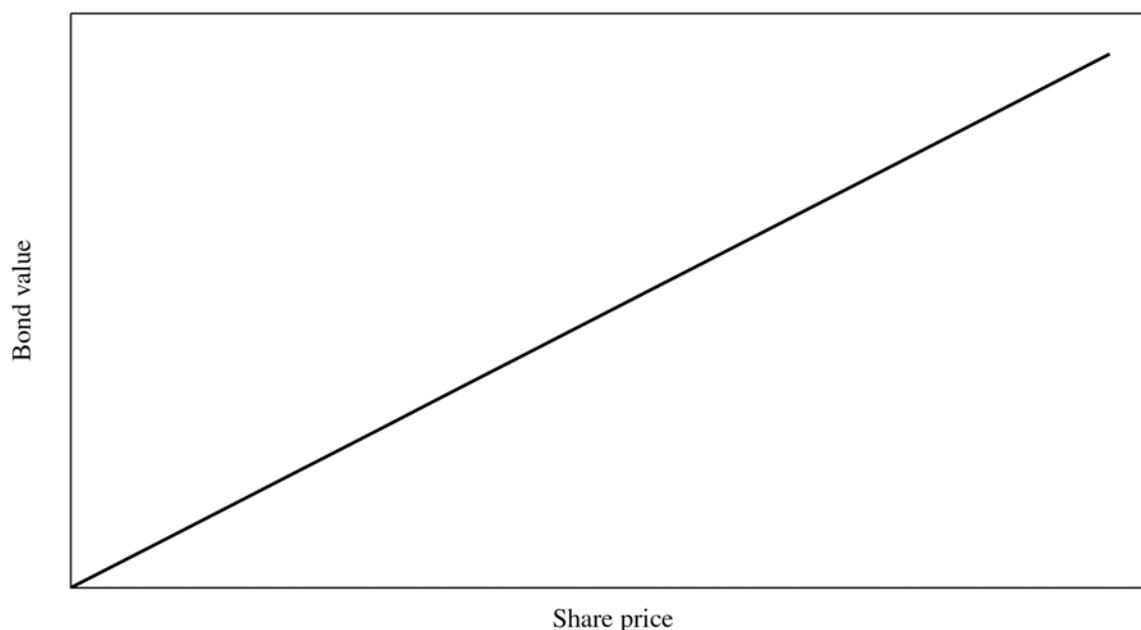


Figure 2: Convertible bond parity line

$$\text{Premium} = \frac{(\text{CB price} - \text{Parity})}{\text{Parity}} \times 100 \quad (4)$$

$$\text{Premium} = \frac{(1000 - 680)}{680} \times 100 = 47.06\%. \quad (4')$$

This implies that the current share price will have to rise by 47.06% for a zero premium to exist, i.e. Parity = (1.4706)(680) = 1000.

The interplay between rising/falling bond and share prices will lead to continually changing parity and premium statistics as time progresses. To illustrate this point and to consider different ways in which a CB can be quoted consider the following example.

Example: CB quoting conventions

Consider a convertible bond offering a coupon of 3.00%, a conversion ratio of 800 shares per nominal £5000 bond at a conversion price of £6.25 per share. The convertible is currently trading at £102.20 (no accrued interest) and the price of the share into which the CB can be converted stands at £5.80. Table 2 shows different ways in which the CB can be valued. Consider the figures in Column 1 first. The nominal value of the CB is £5000 and its conversion price is also £5000. The conversion ratio is given as 800 shares for each nominal £5000 bond. The coupon is 3.00% p.a. and is thus worth £150. The current market price of the bond is £5110. The figure of £5110 can be determined by multiplying the market quote of £102.20 by the CB's nominal value of £5000, and then dividing by 100.

Table 2

	Cash value (1)	Market quote in bond points (2)	Value per share (3)
Nominal value	£5000	100.00	£6.25
Conversion price	£5000	100.00	£6.25
Conversion ratio	800		1
Coupon	£150	3.00	£0.1875
Current price	£5110	102.20	£6.3875
Parity	£4640	92.80	£5.8000
Premium	£470	9.40	£0.5875
Premium (%)	10.13%	10.13%	10.13%

$$\text{Current market price (Cash value)} = \left[\frac{(\pounds 102.20)(5000)}{100} \right] = \pounds 5110.$$

The CB's parity expressed as cash value will be:

$$\text{Parity (Cash value)} = (800)(\pounds 5.80) = \pounds 4640.$$

Premium expressed as cash value will be the difference between the market price of the bond and its cash value:

$$\text{premium (cash value)} = \pounds 5110 - \pounds 4640 = \pounds 470$$

which as a percentage of parity will be:

$$\text{Premium(\%)} = \left[\frac{\pounds 470}{\pounds 4640} \right] \times 100 = 10.13\%.$$

This indicates that the current market price of the share is below the conversion price specified in the CB's term sheet. The investor is paying more for the potential right of buying shares in the company at the specified conversion price some time in the future. Turning now to Column 2, the value of the CB in terms of bond points can be found. This is simply expressing the £5000 unit of trading as a conventional bond quoted off £100 blocks. Reporting a CB in these terms will make direct comparisons with other types of fixed income instruments much easier.

The nominal value of the £5000 CB expressed in bond points will be:

$$\frac{\pounds 5000}{50} \times 100.$$

The conversion price can be treated in the same way. The conversion ratio is 625 for a £5000 unit of trading. It will not have an interpretation if scaled down to 100 bond points so this cell has been left blank. In bond points the coupon will be 3. The coupon is reported as a percentage figure in the bond's term sheet.

The parity figure expressed in percentage terms can be found as follows:

$$\text{Parity} = \left[\frac{(800)(5.80)}{5000} \right] \times 100 = 92.80\%.$$

While the premium in bond points expressed as a ratio of nominal value will be:

$$\text{Premium} = \left[\frac{(5110-4640)}{5000} \right] \times 100 = 9.40$$

and the premium as a percentage of parity will be:

$$\text{Premium} = \left[\frac{9.40}{92.80} \right] \times 100 = 10.13\%.$$

Finally Column 3 provides an insight into the CB's value but reported on a per share basis. The nominal value will be £5000 divided by the conversion ratio. Thus 5000/800 yields a value per share of £6.25. This is the same as the defined conversion price and is quoted on a per-share basis. The cash value of the coupon, £150, when divided by the conversion ratio gives a per-share coupon of £0.1875. The price paid for each share if conversion were to take place now would be £6.3875. Parity is the current market price of the share, £5.80. The difference between the current conversion price and parity provides the premium measure in value terms and this is £0.5875 per share. As a percentage this is once again 10.13% (£0.5875/£5.8000).

If that same bond trades one month later at £100.90 (again – unrealistically – assuming no accrued interest) and the share price has now fallen to £5.40 the revised statistics will be as shown in Table 3.

Note that the fall in the bond's price and the fall in the share price have combined to increase the premium to 16.78%. This is presented in the last row of Table 3.

Table 3

	Cash value (1)	Market quote in bond points (2)	Value per share (3)
Nominal value	£5000	100.00	£6.25
Conversion price	£5000	100.00	£6.25
Conversion ratio	800		1
Coupon	£150	3.00	£0.1875
Current price	£5045	100.90	£6.3063
Parity	£4320	86.40	£5.4000
Premium	£725	14.50	£0.9063
Premium (%)	16.78%	16.78%	16.78%

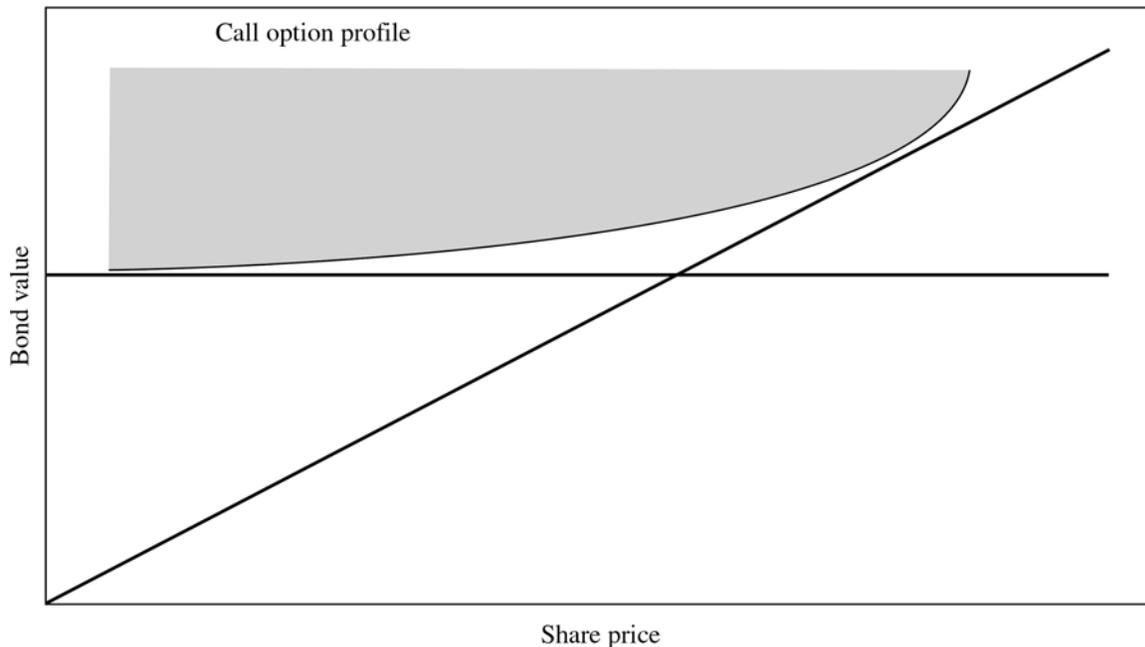


Figure 3: Parity curve with bond floor

The CB floor

A very attractive feature of CBs, from an investor's viewpoint, is the floor that they offer. To illustrate this idea consider a bullet bond redeemable at 100 in 2010. If this redemption value is plotted together with the parity line an interesting feature becomes evident. The shaded area on Figure 12.3 indicates the call option profile created by the CB's parity line and the bond's floor at redemption. The floor in this example is established by the bond's redemption value while the rising portion of the parity line above and to the right of the floor provides the positive payoff emanating from high equity prices. Some CB issues do provide a protection against this eventuality by having the bond underwritten or guaranteed by a third party with a high, or higher, credit rating than the issuer. This added degree of protection is paid for in the form of a higher bond price, which in turn means a lower yield to the bond's holder.

Of course the redemption value will only be realised by the investor should conversion not take place, in effect leaving the bond to be redeemed at maturity for its face value plus final coupon. However, between now and maturity many things can happen. If, for example, the company goes through a bad patch resulting in the price of its stock falling precipitously, the bond price will inevitably suffer. A hypothetical example of this scenario is illustrated in Figure 4 where, as the stock price falls below a certain level the bond's price follows and, in the worst-case scenario, the investor's floor collapses completely as the value of the bond falls rapidly to zero.

In the example above the floor at maturity was illustrated as 100; in practice the CB's floor can be taken to be the investment value of the bond.

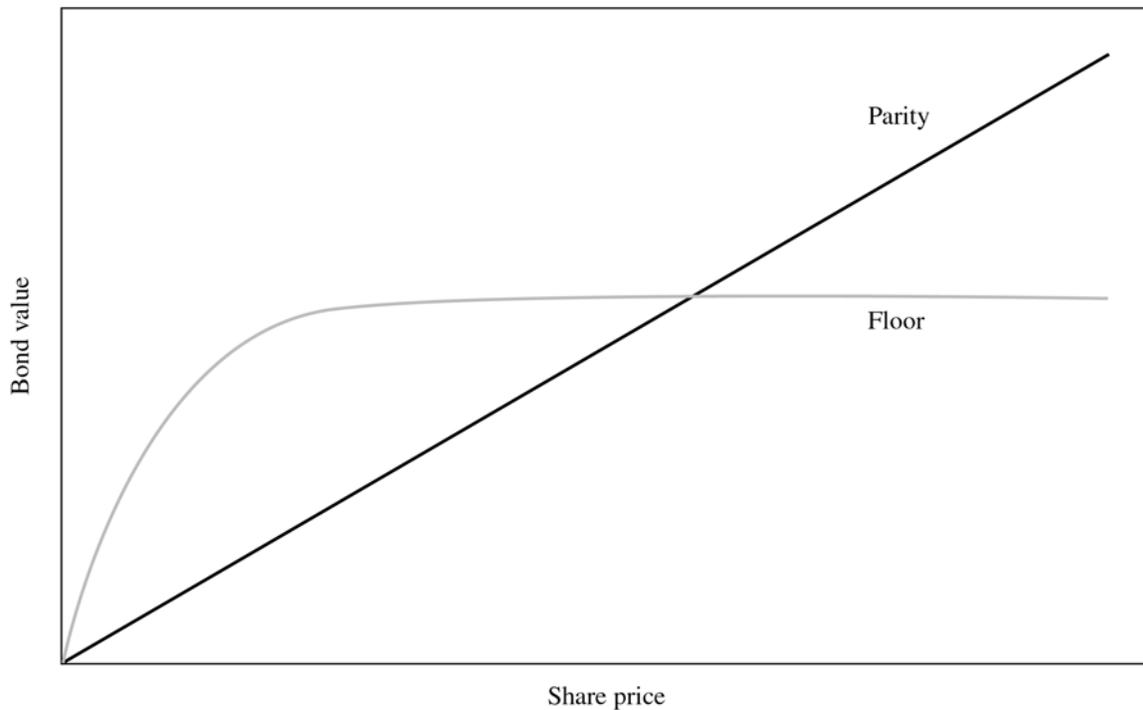


Figure 4: Parity line, CB floor and default

To establish the convertible's floor it is necessary to calculate the straight corporate bond's price and this can be achieved using the familiar bond pricing formula presented in earlier chapters and repeated here for convenience as expression (5).

$$P_o = \sum \frac{C}{(1+r)^t} + \frac{RV}{(1+r)^T} \quad (5)$$

where (for a corporate bond):

- C represents coupon paid annually and fixed throughout the lifetime of the bond;
- r represents the discount rate which includes a risk spread, and r is expressed as a decimal;
- RV represents time to maturity in years;
- t is an index of time in years ($t = 1, 2, \dots, T$);
- T represents the years to maturity.

In reality the CB's floor at any time will depend on interest rates prevailing in the market and will therefore be subject to change even in the absence of company default or threat of default. As an example consider a 5-year, 6% coupon corporate bond priced off a yield-to-maturity of 7.00% plus credit spread of 150 basis points.¹ In the absence of any accrued interest the floor defined by the price of this bond will be £90.15, calculated as shown in Table 4.

¹ More will be said about pricing CBs later.

If, as a result of government policy, benchmark yields rise to 10% and corporate discount rate rises to 11.5% then the level of the floor will fall to around £79.93, calculated as shown in Table 5.

Should the reverse happen with required yield falling to 4% + 150 bps for the corporate bond, the floor will rise to approximately £102.14, calculated as shown in Table 6.

Table 4

Year	Cash flows	Discounted cash flows
1	6	5.52995392
2	6	5.09673172
3	6	4.69744859
4	6	4.32944571
5	106	70.4948149
		90.1483948

Table 5

Year	Cash flows	Discounted cash flows
1	6	5.38116592
2	6	4.82615778
3	6	4.32839262
4	6	3.88196648
5	106	61.507989
		79.9256718

Table 6

Year	Cash flows	Discounted cash flows
1	6	5.68720379
2	6	5.39071449
3	6	5.10968199
4	6	4.84330046
5	106	81.1042415
		102.135142

The instability of the floor can, of course, also be affected by pessimism about the perceived creditworthiness of issuers falling into a particular category or market segment. Such a view could lead to increases in the credit spread component of the discount factor used to value bonds in this credit band.

It is fair to assume that investors must pay for the privilege of owning the long call option feature of a CB. One way in which the cost of this privilege can be assessed is to calculate the extent to which the traded price of the convertible lies above the floor defined by the appropriate straight corporate bond. As an equation this can be shown as presented in equation (6):

$$CB\text{premium} = \frac{(CB\text{price} - \text{Corp.floor})}{\text{Corp.floor}} \quad (6)$$

Thus if, at a point in time, the CB is trading at 102.60 and the bond floor is 84.84 the CB premium will be 20.93%, using equation (6). An important question poses itself at this point, namely: how is the convertible bond to be priced?

Pricing a convertible bond

On occasions in this text reference has been made to the use of trees for instrument pricing. In particular the binomial and trinomial pricing methods have been introduced. An intuitively appealing starting point when analysing the problem of pricing a CB would therefore be to make use of the binomial framework. If we bear in mind certain caveats concerning the use of the binomial and trinomial approaches to pricing, this section will make use of a four-period tree to illustrate CB pricing using this type of lattice approach. The hypothetical convertible bond to be analysed is described in Table 7.

Table 7: Convertible bond: binomial pricing

Number of periods	4
Interest rate (%)	4
Credit spread	1.5
Share price volatility (%)	20
Expiration date (months)	48
Current price of the underlying share	7
Conversion ratio	10.5
Conversion price	9.524
Coupon (annual)	0
Par value of bond (denomination)	100

Thus in this example a 4-year CB is being priced and analysed over the four-year period using one-year time slices. The current risk-free rate of interest is 4%, the bond's coupon is 0%, the par value of the bond is £100, and on conversion each £100 bond can be converted into 9.524 shares of the company at a nominal price of £10.5 per share. The company's share price is currently standing at £7.00, it has a volatility of 20% p.a., and no dividends will be paid over the lifetime of the bond.

Digression: Pricing a bullet bond in a binomial framework

Figure 5 demonstrates how, in principle, a binomial tree can be used to arrive at the price of a coupon-bearing or a zero-coupon bond. Note that a binomial pricing framework is not really required in order to calculate the price of a straight corporate bond – unless trying to establish an appropriate implied credit spread using option adjusted spread analysis is the object of the exercise.² In the present example, however, it is assumed that the credit spread is known and is given as being 150 bps above the risk-free rate.

Thus using 5.50% (4.00% + 1.50%) as the appropriate discount rate for each node and working backwards applying the p and $(1 - p)$ parameter values at nodes K, L, M, N, O through to A the price of the bond is found to be 80.72.³ This value represents the CB's floor. Since the CB in Table 7 is a zero-coupon bond this price could quite easily have been found using the standard pricing approach:

$$100/(1 + 0.055)^4 = 80.72$$

or, in the case of a coupon-bearing bond by using the pricing approach adopted when establishing the hypothetical floors which appear in Tables 4 to 6.

Using the up (u), down (d) approach of the binomial framework, the parameter values p and $(1 - p)$ are calculated as 1.2840, 0.7788, 0.4879, 0.5120, respectively. These parameters appear in Figure 6. Using the u and d values the progress of the company's share price can be calculated at each node of the tree.

Figure 6 illustrates a constant credit adjusted spread method of pricing the CB. The discount factor used here is the risk-free rate 5.50% (4.00% risk free rate + 1.50% credit spread) and is applied to each node in the tree. Starting again at the maturity end nodes (K, L, M, N, O), the procedure involves comparing the bond redemption value plus final coupon against parity with the larger of the two values being adopted as the price of the CB at that node. The parity values are found in the usual way by applying u (up) and d (down) values calculated according to the formulae presented in the discussion of the binomial methodology in Chapter 11. Here u equals 1.2214 and d equals 0.8187. The values £7.00 (node A), £8.55 (node B), £5.73 (node C), £10.44 (node D), £7.00 (node E), £4.69 (node F), etc. trace out the possible paths followed by the share price given the input data.

At each final node at the end of year 4 a comparison is made between the maturity value of the bond and the value associated with converting the bond into shares. In this case each bond can be converted into 10.5 of the company's shares so that at each individual node conversion would result in a total value of final share price multiplied by 10.5. The larger of the two figures at each node is taken to be the end value for that node. To clarify this point consider node K where the final share price is calculated via the tree as £15.58.

² For a description of this process see Eales, B. A., *Financial Engineering*, Palgrave (2000).

³ To arrive at the figure of 94.79 which appears at nodes G, H, I, J , the end nodes K, L, M, N, O (100) are discounted by 1.055. Any coupon due at the end of year 3 is then added to the result. The same steps are followed for nodes D, E, F , etc. until the end node price of £80.72 per £100 nominal is obtained at the start node A .

On multiplication this generates a figure of £163.58 while the bond's redemption value plus final coupon will be £100.

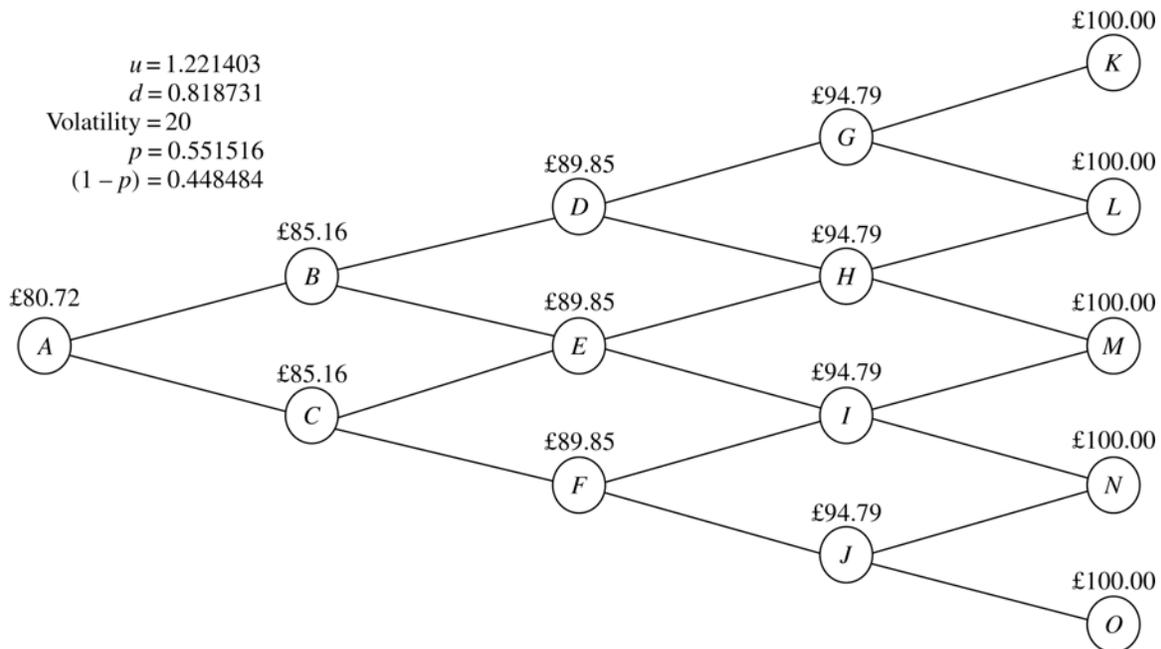


Figure 5: Bullet corporate bond

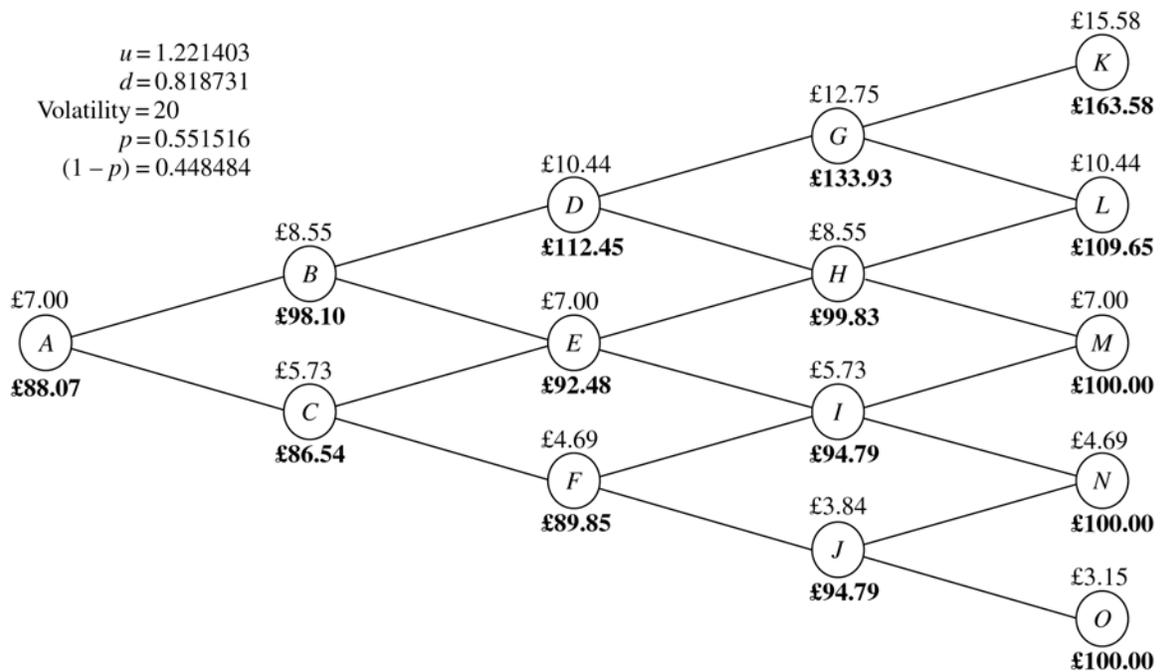


Figure 6: Convertible bond priced off constant credit spread

Assuming that a rational decision will be taken by the investor, the figure adopted as the end node value will clearly be £163.58. In other words the bond will be converted. The same will hold true for node *L*. Since £109.65 is greater than the £100 redemption value of the bond conversion will take place. At nodes *M*, *N* and *O* the bond will be redeemed for par value plus final coupon. At some points where redemption value plus coupon and terminal value of shares are very close the decision to convert may become more complex. Institutional and transaction costs, the dilution factor and investor preferences may come into play and influence the final decision.

Once these end node values have been established the familiar process of sliding back through the tree is used to arrive at values for nodes *G*, *H*, *I* and *J*, in the first instance. Applying the estimated parameters ($p = 0.5515$ and $(1 - p) = 0.4485$) to the values at nodes *K* (£163.58) and *L* (£109.65) a value of £133.93 is found. This value needs to be discounted at the credit adjusted rate of interest (5.50%), i.e. $£133.93/1.055 = £132.12$ to obtain the present value of the convertible bond. This value must now be compared to the value of the shares at that node: $(10)(£12.75)$. From node *G* in Figure 6 an accurate value for the total value of shares is £133.92. Notice that no coupon needs to be added since the CB is a zero-coupon bond. The value of the CB at that point is thus parity £133.92 since it is greater than the backward calculated bond price of £132.12. Node *H* is found in a similar way but now taking £109.65 and £100.00 as results to be weighted and discounted. This yields £99.83 which compared to parity of $(10.5)(8.55) = £89.77$ would lead to a decision to hold on to the bond.

Repeating this process back to nodes *B* and *C* the values of £98.10 and £86.54, respectively, are obtained. By inserting these values in equation (6) a CB price of £88.07 is obtained.

$$CB\ price = \frac{[(98.10)(0.5515) + (86.54)(0.4485)]}{1.055} = 88.07 \quad (7)$$

Philips (1997) suggests, however, that this method of pricing a CB will generally lead to mispricing. He argues that when the company's share price is high and the CB is highly in-the-money the risk of default will be minimal and therefore a more appropriate discount rate to apply would be the risk-free rate with the basis point credit risk add-on being dropped. To facilitate the incorporation of this factor he proposes a delta-based technique that effectively adjusts the discount rates used at each node. The formula he advances appears as equation (8).

$$r^* = \Delta r_1 + (1 - \Delta)r_2 \quad (8)$$

where

- r^* represents the delta adjusted discount rate;
- r_1 represents the risk-free rate;
- r_2 represents the credit-adjusted rate;
- Δ represents the rate of change of the CB price/rate of change of parity.

The automatic question that then arises is how can the delta be calculated at each node. One approach would be to use the standard formula for calculating a hedge ratio (h) for written calls (puts), namely:

$$h = \left(\frac{C_u - C_d}{S(u - d)} \right) \text{ for calls or } h = \left(\frac{P_u - P_d}{S(u - d)} \right) \text{ for puts.}$$

An alternative method would be to perturbate the starting price of the share by some small amount and find the small finite change that has occurred to the parity and bond figures in the binomial tree. This can be represented as follows in equation (9):

$$\text{CB delta} = \frac{\partial \text{CB}}{\partial \text{Parity}}. \quad (9)$$

Either approach allows the discount rates to be adjusted to reflect the in- or out-of-the-moneyness of the bond. In the example illustrated below using the method of tree perturbation and the data in Table 8 (which for convenience has been rounded to three decimal places) the delta at node A is found to be 0.533.

The change in the convertible bond's price when the price of one share is reduced by 0.01 to £6.99 is £88.015. This compares to the £88.071 established for a share price of £7.00 using the binomial tree. At the same time the change in parity is 0.105 [10.5(7.00–6.99)]. Delta can thus be calculated by subtracting the perturbed pass value of the bond from the originally calculated bond price and dividing this result by the change that has taken place in parity. Equation (12.9') demonstrates this approach for node A :

$$\left[\frac{88.071 - 88.015}{73.50 - 73.395} \right] = \left[\frac{0.056}{0.105} \right] = 0.533. \quad (9')$$

All other deltas can be calculated in the same way.

Table 8: Bond price and parity changes

(a) First pass									
Bond price	Parity	Bond price	Parity	Bond price	Parity	Bond price	Parity	Bond price	Parity
163.577	163.577	133.926	133.926	112.450	109.649	98.099	89.773	88.071	73.500
109.649	109.649	99.831	89.773	92.482	73.500	86.540	60.177		
100.000	73.500	94.787	60.177	89.845	49.269				
100.000	49.269	94.787	40.338						
100.000	33.026								
(b) Second pass									
Bond price	Parity	Bond price	Parity	Bond price	Parity	Bond price	Parity	Bond price	Parity
163.344	163.344	133.734	133.734	112.315	109.492	98.011	89.645	88.015	73.395
109.492	109.492	99.749	89.645	92.439	73.395	86.517	60.091		
100.000	73.395	94.787	60.091	89.845	49.198				
100.000	49.198	94.787	40.280						
100.000	32.978								

The deltas calculated using the perturbed tree method are reported in Table 9. Once the deltas for each node have been calculated the revised discount rates can be estimated. For node *K*, for example, the delta-adjusted discount rate will be:

$$r^* = (1) (4.00\%) + (1 - 1) (5.50\%) = 4.00\% \quad (8')$$

whereas for node *H* the delta adjusted rate will be:

$$r^* = (0.641) (4.00\%) + (1 - 0.641) (5.5\%) = 4.5385\% \quad (8'')$$

Figure 7 demonstrates the impact that this approach will have on a CB's price. Using the delta adjusted rates as discount factors at each relevant node the price of the CB is found to be £90.61 compared to the £88.07 price obtained using more conventional methods. Of course these findings are based on a four-period tree which will not reflect the CB's true value, nevertheless when price differences exist in bi- or trinomial trees with many branches an arbitrage opportunity may well present itself. Since the premium calculated using the straightforward binomial model appears high compared to the adjusted discount approach the CB can be synthesised by buying an appropriate quantity of the company's equity. To complete the process, sell the CB. In this way a position is engineered which will ensure an arbitrage profit.

Table 9: Binomial tree deltas

Delta	Node								
1.000	K	1.000	G	0.860	D	0.688	B	0.533	A
1.000	L	0.641	H	0.410	E	0.267	C		
0.000	M	0.000	I	0.000	F				
0.000	N	0.000	J						
0.000	O								

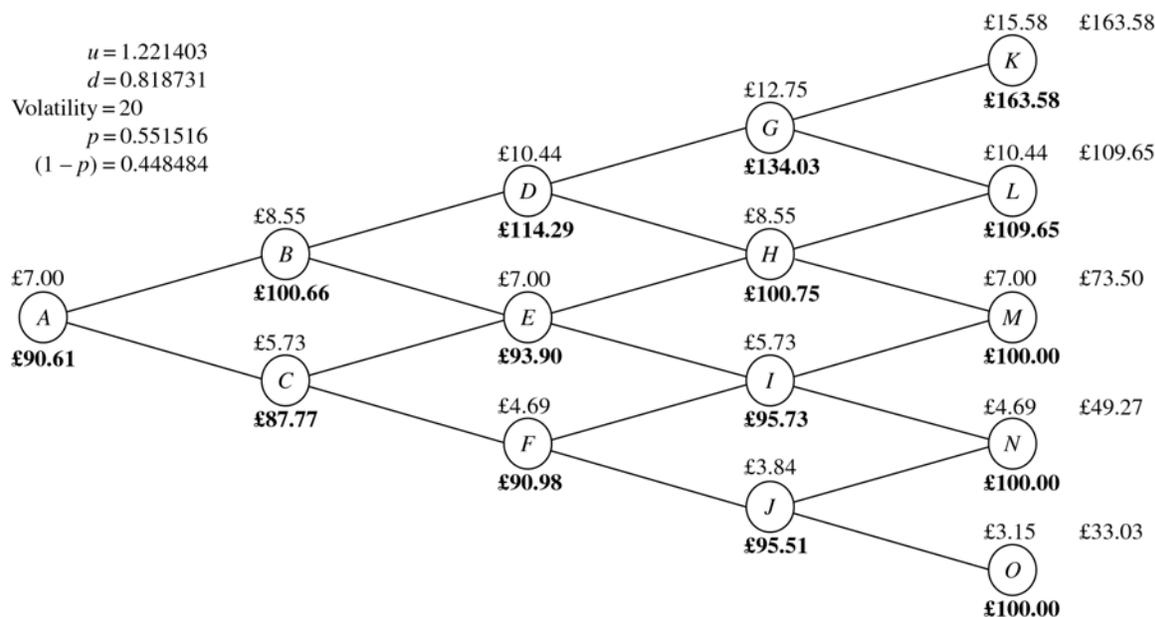


Figure 7: Convertible bond priced off constant credit adjusted rates

Turning to the trinomial pricing approach a similar pattern emerges. Figure 8(a) depicts a trinomial tree where the bond has been discounted at a constant 5.5% rate at each node. The price derived using this method is £87.90. Whereas using the approach suggested by Philips, and a lambda set arbitrarily at 3, Figure 8(b) reveals a price of £90.32. The empirical deltas used to estimate this adjusted discount rate price and the adjusted rates themselves found using the perturbation method are shown in Tables 10 and 11, respectively.

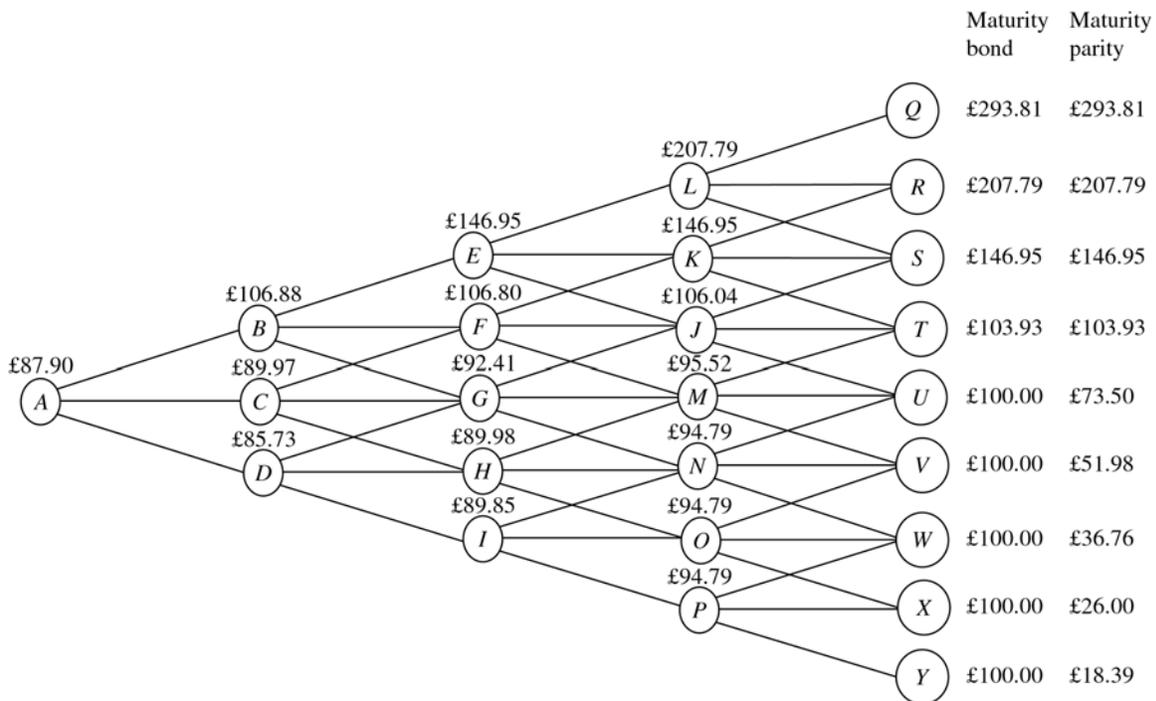


Figure 8(a): Convertible bond priced off constant credit spread

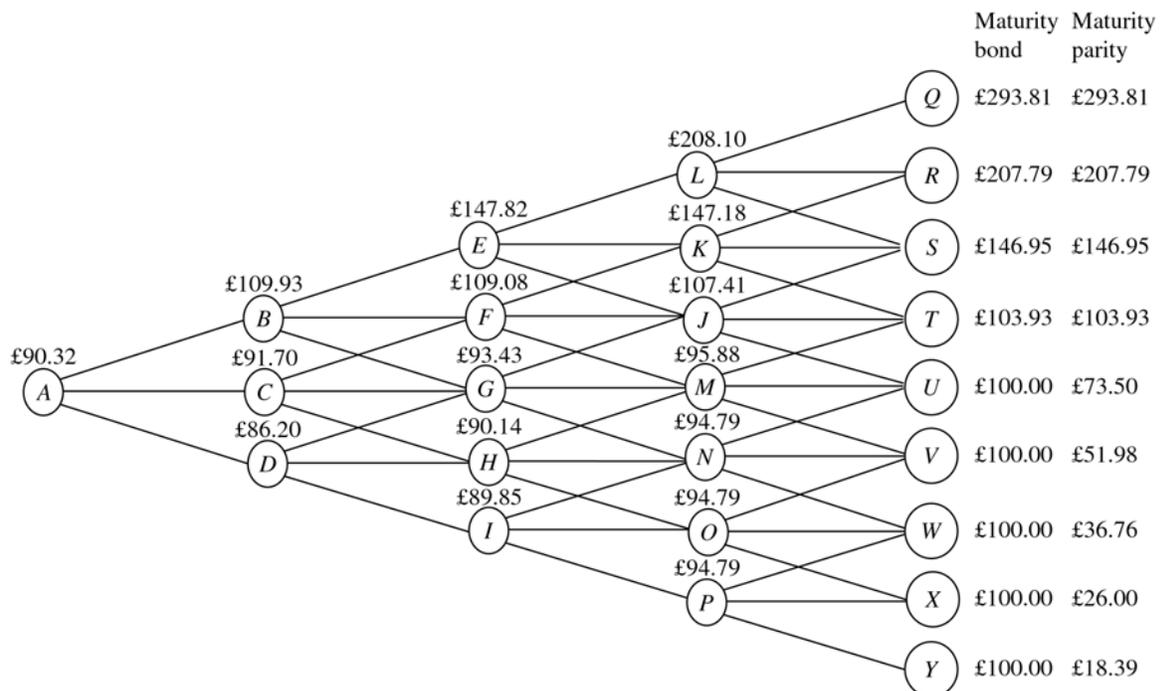


Figure 8(b): Convertible bond priced off delta adjusted interest rates

The reader should note caveats concerning pricing using lattice frameworks. Here the results presented are for illustrative purposes. To gain an accurate price far more branches would need to be incorporated into the tree, and for the trinomial tree an appropriate value for lambda would need to be established.

Additional CB features

Another interesting modification that can be introduced is the provision for a downward reset, or refix, minimum conversion rate at some specified future date. This feature is incorporated into a CB as a means of forcing conversion: an additional amount of the company's stock will be created but the bond will not be redeemed for cash. An example of this type of refinement is provided by the Mitsui Trust and Banking 0.5% CB, 1 August 2007. Provision is made for this bond to be converted at ¥920 with a minimum conversion price of ¥500. Figure 9 depicts the general effect that a reset will have on the instrument's profile. It should be noted that the effect of a reset clause may give rise to negative convexity in the CB's payout profile prior to expiration. The reset feature resembles an option whose strike price is reset when predefined conditions have been satisfied, for example, ladder options, or in the case of specified future dates cliquet options described earlier. The diagram also provides a reminder that as stock price falls the bond will also decline and may trade at a discount to par.

Table 10: CB bond price and parity changes

Period 4 Bond	Parity	Period 3 Bond	Parity	Period 2 Bond	Parity	Period 1 Bond	Parity	Period 0 Bond	Parity
293.808	293.808	205.184	207.787	145.384	146.952	106.884	103.928	87.895	73.500
207.787	207.787	145.111	146.952	106.805	103.928	89.972	73.500		
146.952	146.952	106.045	103.928	92.414	73.500	85.728	51.981		
103.928	103.928	95.521	73.500	89.982	51.981				
100.000	73.500	94.787	51.981	89.845	36.762				
100.000	51.981	94.787	36.762						
100.000	36.762	94.787	25.999						
100.000	25.999								
100.000	18.387								

Period 4 Bond	Parity	Period 3 Bond	Parity	Period 2 Bond	Parity	Period 1 Bond	Parity	Period 0 Bond	Parity
293.388	293.388	204.891	207.491	145.179	146.742	106.759	103.779	87.838	73.395
207.491	207.491	144.903	146.742	106.678	103.779	89.921	73.395		
146.742	146.742	105.912	103.779	92.371	73.395	85.717	51.907		
103.779	103.779	95.493	73.395	89.977	51.907				
100.000	73.395	94.787	51.907	89.845	36.709				
100.000	51.907	94.787	36.709						
100.000	36.709	94.787	25.962						
100.000	25.962								
100.000	18.361								

Table 11: Delta adjusted discount rates

Adjusted rate	Node	Adjusted rate	Node	Adjusted rate	Node	Adjusted rate	Node	Adjusted rate	Node
0.0400	Q	0.0402	J	0.0403	E	0.0424	B	0.0468	A
0.0400	R	0.0402	K	0.0422	F	0.0477	C		
0.0400	S	0.0416	L	0.0489	G	0.0527	D		
0.0400	T	0.0510	M	0.0540	H				
0.0550	U	0.0550	N	0.0550	I				
0.0550	V	0.0550	O						
0.0550	W	0.0550	P						
0.0550	X								
0.0550	Y								
Risk-free credit spread adjusted rate		0.04							
		0.055							

From the point of view of the investor downward resets may be attractive if they anticipate share prices to perform badly over the medium term but to recover by the time that the bond reaches maturity or is called. The investor then enjoys acquisition of shares at a cheaper rate but the issuer will need to issue more stock and will decrease the influence of existing equity holders. This is known as the dilution effect.

Upward resets are also issued and operate in the opposite way to the downward reset feature described above. This type of clause leads to the likelihood that the CB will be redeemed rather than converted but will also mitigate the dilution effect if conversion does take place and new shares are issued.

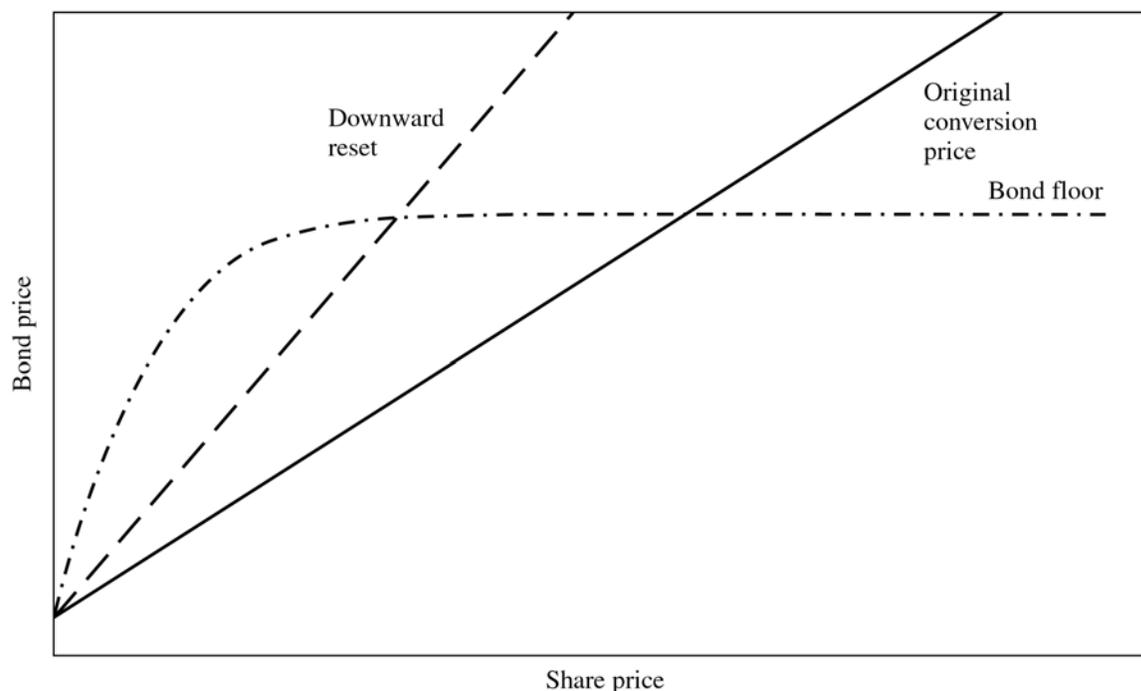


Figure 9: Reset convertibles

Embedded calls

Convertible bonds are often issued with a call feature that enables the issuer, if they so wish, to force the holder to convert when the company's share price reaches a certain level. Frequently the call provision is activated or triggered when the share is 30% above the conversion price. So with a conversion price of £10 the bond might be deemed callable when the price rises above £13 and stays at above that level for more than a specified period of time. Whether or not the bond is callable will depend upon any conditions that were specified in the bond's original indenture. To protect the holder of a CB from such an eventuality provisions are normally made in the indenture to the effect that the bond cannot be called in the first x years following issue. This stipulation affords some protection to the holder of the bond and is referred to as *hard non-call* or *hard call*. Once the x years have elapsed the bond moves into a *soft non-call* or *soft call* period. The bondholder now faces the risk that the bond may be redeemed once the call level defined in the indenture has been reached. To offer the holder protection against market "spikes" there may be an additional condition to the effect that the bond cannot be called unless it has traded at, or above, the defined trigger call level for a specified number of days. The protection afforded by the hard call period ensures that the bondholder should, in the majority of cases – ignoring the risk of default – receive a coupon advantage over the holder of equity since coupons are usually higher than dividends.

If all issuers decided to call their bonds once the conditions in the indenture had been met the effect would be to form a collar as demonstrated in Figure 10. In practice, for a variety of reasons, convertible bonds are not always called once the trigger has been activated. Figure 11, which ignores the possibility of a collapsing floor, illustrates a traded bond's potential profile before the bond reaches maturity. At low share prices the bond floor will provide the bond price. At slightly higher prices the CB price rises above the floor but once the share price has exceeded some critical level the CB price will be drawn in towards the parity line. This is, of course, the typical behaviour of a standard call option prior to expiration.

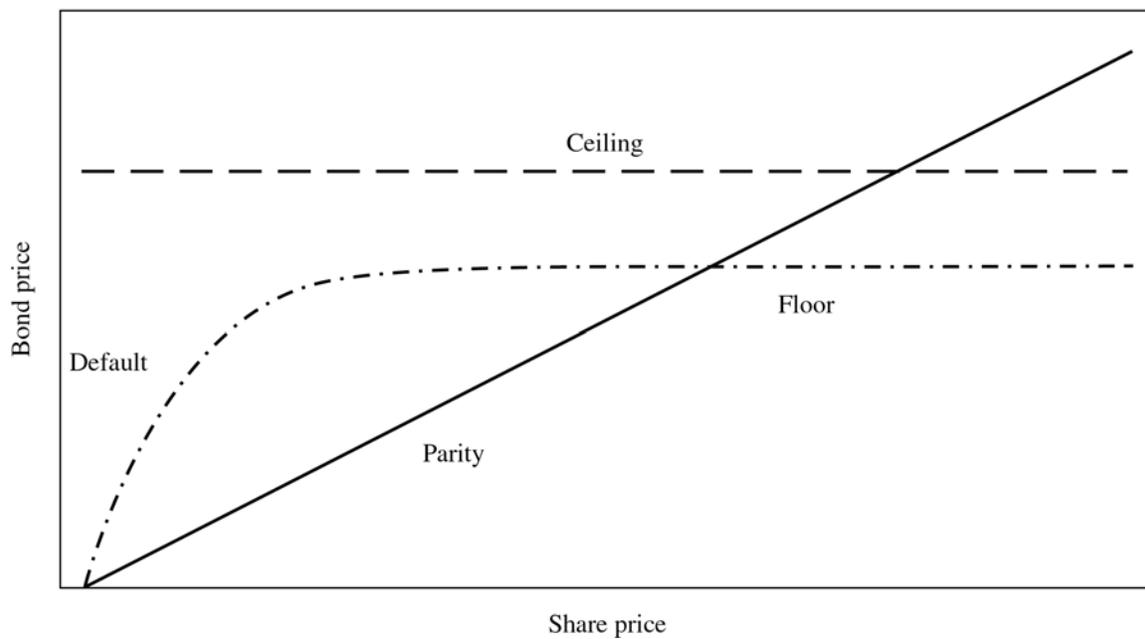


Figure 10: CB with embedded call option

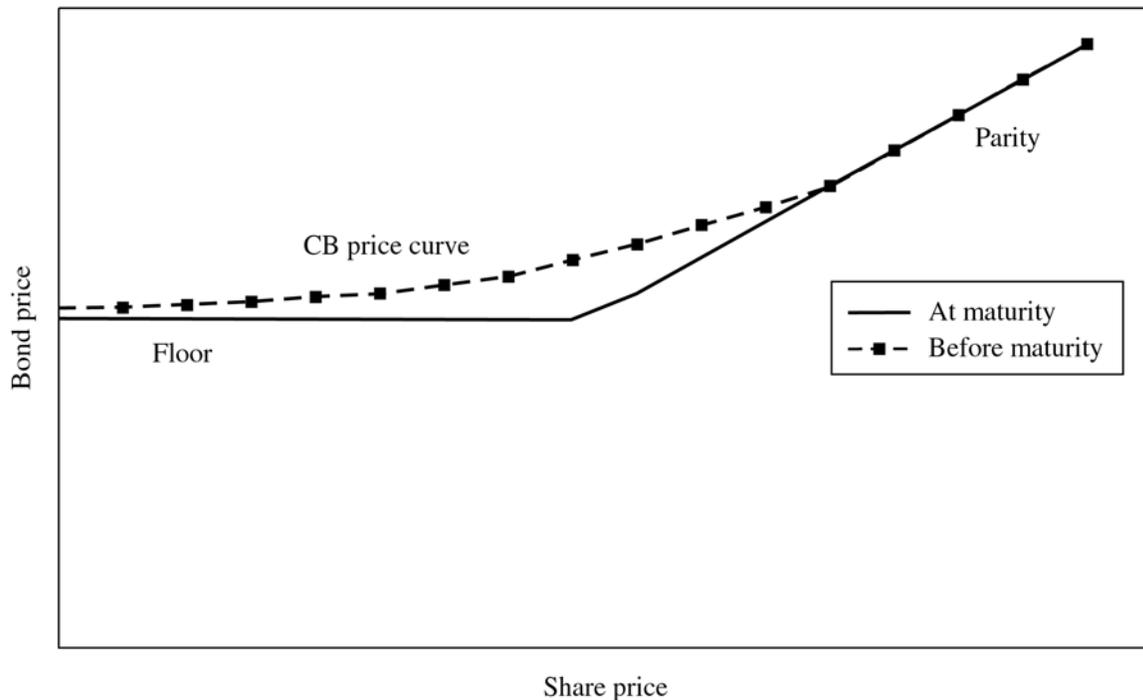


Figure 11: Convertible bond price behaviour

Pricing a CB with an embedded call is relatively straightforward in a binomial framework. Take for example the CB if the bond houses a call feature, say, callable at end of year 3 and the soft call period has been entered. If the bond price at any node is greater than parity then the price of the bond must be reduced to parity. The logic behind this is that since the bond is callable and the call provision in the bond's indenture has been met, the bond may be withdrawn from the market by the issuer at any time. To clarify this, consider the following example.

If parity is currently £142, say, and the bond price is estimated in the tree as £144, the bond should trade at £142. If it trades at £144 and is called, the holder will convert and the bond will cease to exist. Knowing this the potential purchaser of the bond should not be willing to pay an amount that would be received back in the form of shares if conversion is forced as a consequence of the issue of a call notice.

Applying this idea to the credit adjusted spread tree depicted in Figure 7, if the bond is callable at £105 from the end of year 2 when the price of the bond trades at £105 or higher, nodes *G* and *K* will need to be cut from the tree to establish today's fair price. Taking this change into account, suitably modifying the tree, and recalculating the deltas the revised convertible bond price can be calculated. The effect of the call is to reduce the CB's price to £88.03. In effect the bondholder has sold the call option and the bond's price is reduced by the call premium.

As discussed above, although convertible bonds may be callable there is no guarantee or compulsion on the part of the issuer to issue a call notice. Many issues trade well above their call trigger levels. In this event the bond price will continue to rise as the share price rises and it is parity that defines the cap on the price of the CB under those circumstances.

Embedded puts

This type of instrument conveys the right but not the obligation to return the CB to the issuer at a price defined in the indenture at a specified point in time. Clearly, now the lower branches of the pricing tree become important when making the decision to exercise the put. However, lopping the lower CB values from the tree will have the effect of increasing the bond's market price. In this case the issuer is selling a put option to investors and that privilege will have to be paid for.

Mandatory conversion structures

Apart from the problem of pricing CBs the bond/equity mix embodied in a convertible leads to other difficulties. From an external perspective, how, for example, should a convertible be treated by the relevant regulatory authorities: is the instrument a bond generating a series of period cash flows which may be tax deductible or is it equity and part of the company's permanent capital base? How should financial market analysts regard the instrument when advising on investment strategies? From the company's internal perspective the big question concerns whether or not conversion takes place. To some extent the inclusion of reset clauses, discussed above, may go some way to ensuring that a CB is converted at maturity, but conversion is by no means certain; for this reason some convertible bonds have mandatory conversion stipulated in their contract specification. There are several types of structures that fall into this category:

1. Equity commitment notes – where repayment is effected through a special fund established from moneys obtained through the sale of equity securities that qualify under accounting practices as primary capital.
2. Equity notes – where repayment is undertaken through either an optional fund created in the same way as described in 1 or by physically delivering equity from the stock of primary capital.

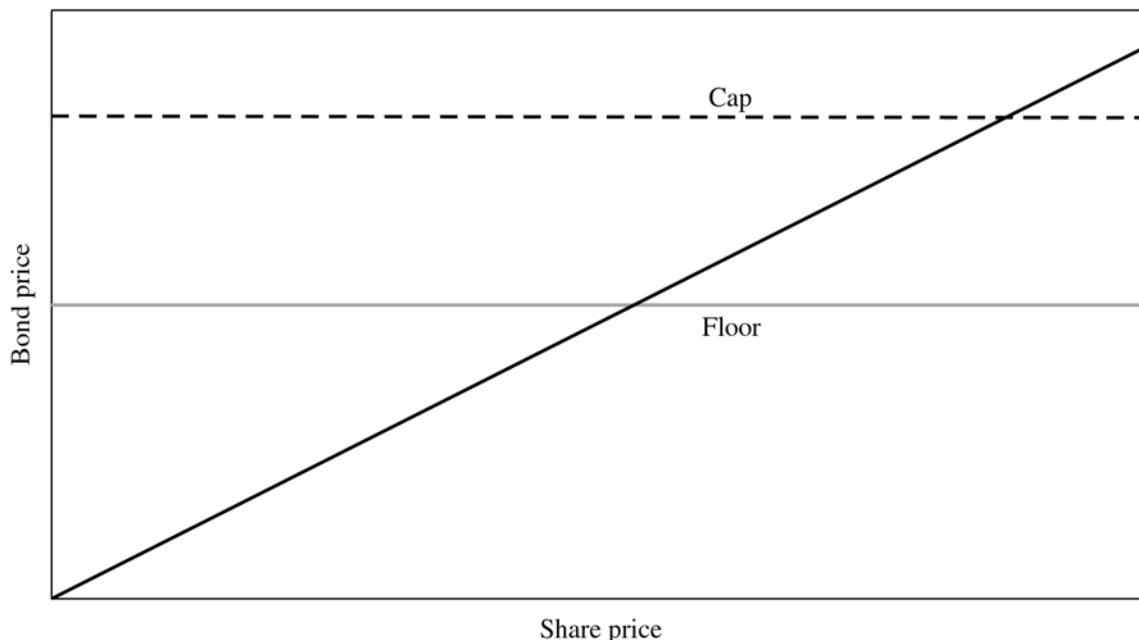


Figure 12: Convertible bond with cap and floor

Some mandatory conversion structures also have caps built into them. Typical in this category would be the PERCS (preferred equity redemption cumulative stock), ELKS (equity-linked debt securities) and EYES (equity-linked yield securities). The examples cited do not exhaust the mnemonics created by the financial engineering and structural product creators.

In the case of these mandatory conversion issues, the conversion rights section of the contract specification will be phrased to reflect the issuer's concern with potential dilution problems. The wording may take the form: "The Bond can be converted into the Issuer's shares subject to the defined conversion cap and floor. However, the Issuer reserves the right to cash settle the transaction." An example of a mandatory CB with capped share price of 132 and floor share price of 100 is illustrated in Figure 12.

The valuation of CBs with these complex structures can be handled by the tree methods already described. Another possibility, however, is to synthesise the product. In the case of a capped mandatory convertible this would entail the purchase of the underlying shares and, simultaneously, the writing of an OTM call option on the shares with an exercise price set at the level of the specified cap and with the same maturity date as the CB. When a floor is included in the transaction this would also require the simultaneous purchase of a put option on the shares with an exercise price equal to the specified floor level and, again, with a horizon matching that of the maturity date of the CB.

Reverse convertibles

This type of CB is engineered so that at maturity it is either converted into shares if the underlying share price multiplied by the conversion factor is *less than* the redemption (par) value of the bond or redeemed at its par value plus final coupon. This decision rule is clearly the exact opposite of that used in the analysis of the CBs discussed in this chapter. It is thus the issuer of the bond who has the possibility of deciding at maturity which redemption strategy is the more economic.

This instrument creates a position that (from an investor's perspective) is the same as having purchased a fixed-coupon bond and having written a European-style put option on the issuing company's equity. The instruments do not really offer equity or bond fund managers the features of the normal convertible bond. From the point of view of equity participation the upside potential of a reverse CB is capped and, moreover, it offers no supporting floor on the downside. On the plus side these instruments will offer coupon rates higher than those enjoyed by similar quality paper.

Currency considerations

This chapter has attempted to demonstrate the basic construction and pricing of convertible bonds. The focal point has been on single currency CBs and some modifying features that have been introduced in recent years. One further aspect, which adds another complicating dimension to the pricing of these instruments, but may also enhance their attractiveness to issuers and/or end users, is currency. Convertible bonds are often issued with features that involve more than one currency. In the indenture, for example, the shares may be denominated in one currency but the bond redeemable in another. Under such a regime the bond's price will reflect not only movements in yields but also movements in the foreign exchange markets. It is clear that this type of instrument brings with it additional risks as well as potential gains. A good exposition of the way in which such convertibles can be priced appears in Connolly (1998).

Convertible bond credit spreads

The main concentration of this chapter has been on the equity and bond characteristics that the instrument offers. The time has now come to examine credit aspects. This can be achieved by breaking down the bond into its component parts and structuring an asset swap to handle each identified exposure.

Scenario 1

A 5.00% convertible bond with two years to maturity is purchased and held in a portfolio by a fixed income fund manager attracted by the possibility of gaining a relatively safe investment but with the good upside potential offered by the equity component of the bond. The fund manager realises at this point that the equity component has performed well since its purchase and he/she wishes to lock into the equity gains without converting. This "locking-in" could be achieved by writing a call option and buying a put option with a strike price that reflects the current price of the underlying equity. If exchange-based options are available they could be used but in their absence OTC options would be a substitute. Alternatives to this would be to short a USF – if available – or use Contracts for Differences. Up to a point this effectively covers movements up or down in the value of the share price leaving the overall position flat. Figure 13 illustrates how this position is hedged. It should be noted that should the underlying share decline below the floor of 97.5 the short position will continue to generate profits that will augment those obtained from the CB's floor. This, of course, raises the possibility of reducing the cost of the lock-in strategy by writing a put option with a strike in the region of 97.5. Ignoring this last possibility there are a number of possible outcomes to this strategy. These are summarised in Table 12.

Scenario 2

An investor has purchased a CB issued by a company with a sound trading background and whose bonds are regarded as investment quality. The credit spread on the CB appears wider than what would normally be reasonable for an issuer of this standing. The investor writes a call option on the equity component of the CB, which will be activated when the company's share price reaches a certain level. At this point in the strategy the investor holds a synthetic callable bond. There are several outcomes possible and these are summarised in Table 13. There are many other strategies available using CBs but those in Tables 12 and 13 provide a flavour of what is possible.

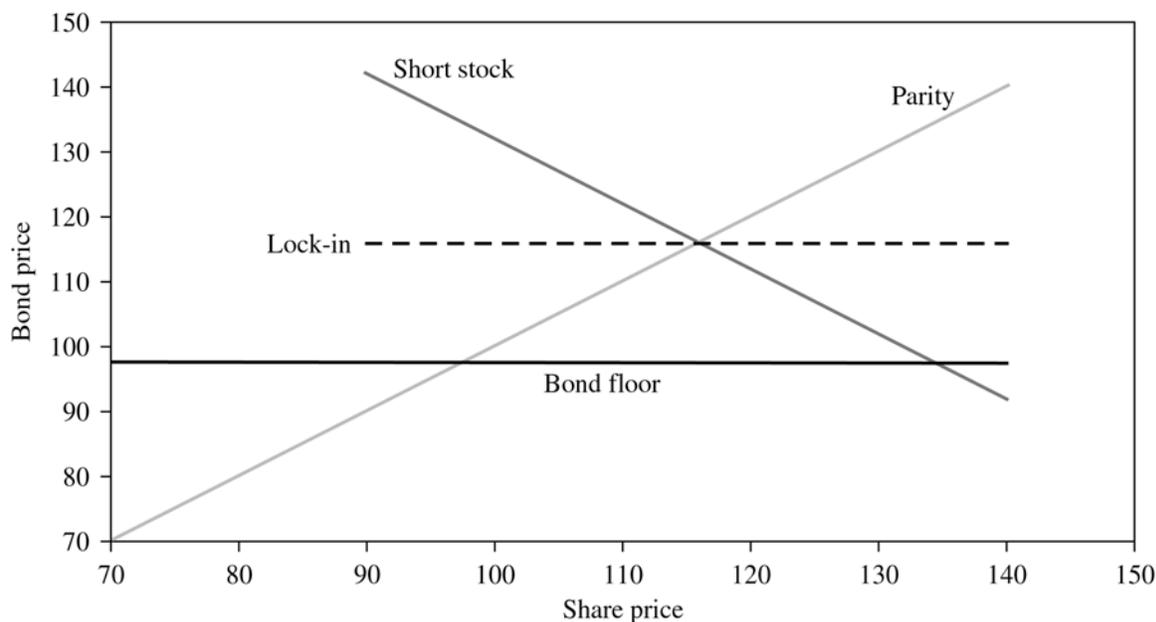


Figure 13: Equity lock-in

Table 12

Outcome	Action
CB reaches maturity and is in-the-money	Closes the “short” position (if the market price of the equity is below the locked-in price the “short” position will deliver a profit. If the market price of the equity is above the locked-in price the “short” position delivers a loss). Converts CB at the conversion price and sells equity at market.
CB reaches maturity out-of-the-money	Closes the “short” position. Since the price of equity has fallen the “short” position will deliver a profit to ensure the locked-in value. Receives the bond’s par redemption value.
CB is called in-the-money	Closes the “short” position at parity. Receives the call price from the issuer.
CB is called out-the-money	Closes the “short” position and receives a positive payoff to ensure locked-in price is achieved. Receives the call price from the issuer.
Issuer defaults on CB	Closes the “short” position. Receives a positive payoff to ensure locked-in price is achieved. Receives the recovery value from the bond issuer.

Table 13

Outcome	Action
CB reaches maturity and is in-the-money	Call option is exercised by the holder. Investor converts into equity at the conversion rate and sells equity at market. The investor receives par and passes equity gain to the option holder.
CB reaches maturity out-of-the-money	Call option expires worthless. Receives indenture-specified redemption value from the issuer.
CB is called in-the-money	Call option is exercised by the holder. The investor receives indenture-specified call value and passes equity gain to the option holder.
CB is called out-of-the-money	Call option expires worthless. Investor receives indenture-specified call value from the issuer.
Issuer defaults on CB	Receives the recovery value from the bond issuer.

* * *

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